DOCUMENT RESUME

ED 372 921 SE 054 176

AUTHOR	Flewelling, Gary
TITLE	Math Activities Using LogoWriter--Investigations.
INSTITUTION	International Society for Technology in Education, Eugene, OR.
REPORT NO	ISBN-1-56484-034-4
PUB DATE	93
NOTE	90p.; For related documents, see SE 054 175-177. The "LogoWriter" application is not included in the package and is necessary for using the disk files.
AVAILABLE FROM	International Society for Technology in Education, 1787 Agate Street, Eugene, OR 97403-1923 (Package includes this book, a data disk, and a single-user site license; disks are available for the Apple, MS-DOS, and Macintosh platforms).
PUB TYPE	Guides - Classroom Use - Teaching Guides (For Teacher) (052)
EDRS PRICE	MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS	Arithmetic; Computer Assisted Instruction; Decimal Fractions; Diagrams; Elementary Secondary Education; *Geometry; *Mathematics Instruction; *Polygons; *Problem Solving; Visualization; Worksheets
IDENTIFIERS	*Computer Integrated Instruction; Conjectures (Mathematics); Greatest Common Factor (Mathematics); *LOGO Programing Language; Mathematics Activities

ABSTRACT

This book is one in a series of teacher resource books developed to: (1) rescue students from the clutches of computers that drill and control; and (2) supply teachers with computer activities compatible with a mathematics program that emphasizes investigation, problem solving, creativity, and hypothesis making and testing. This is not a book about Logo, but about mathematics and thinking mathematically. Many chapters require no Logo skills. Each chapter may include as many as seven sections: summary box, getting started, instructions, examples, activities, key summary, and worksheets and includes appropriate grade-level designations. The chapters in this book are: (1) DeDot Designer, (2) Spirolatera Designs, (3) Digital Root Patterns, (4) GCF and Line Designs, (5) Two Frogs and a Frisbee, (6) Repeating Decimals, (7) Super Wriggly Worms, and (8) Superpoly. Appendices include a summary of LogoWriter keys (Apple IIe/IIc, IBM, and Macintosh), a quick reference (LogoWriter Primitives and Special Words), and additional primitives for the Macintosh. (MKR)

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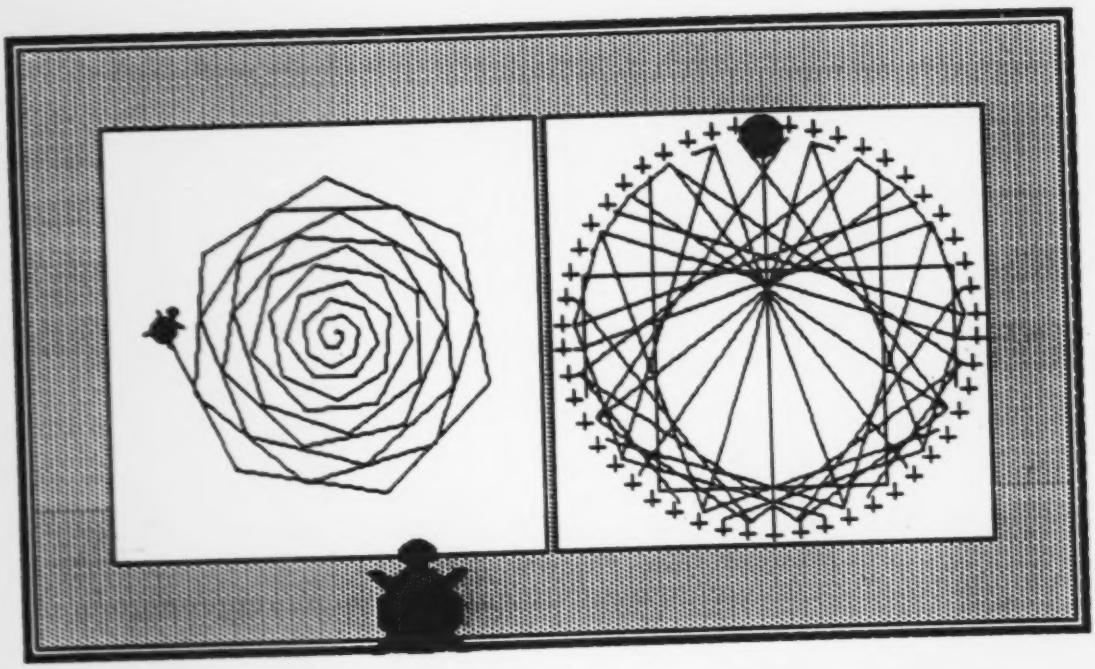
ED 372 921

Math Activities Using *LogoWriter*

Investigations



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About the Author

Gary Flewelling received his B.Sc. from the University of Windsor, Windsor, Ontario, Canada, and his M.M. (Master of Mathematics) from the University of Waterloo, Waterloo, Ontario, Canada. He has been a teacher since 1964, and a mathematics consultant for the past 13 years.

His main interests in mathematics education are natural learning and the integration of technology in the math classroom. He developed—and is still developing—*Math Activities Using LogoWriter* products and books out of a belief that computers should be used more effectively in the math classroom and that Logo should be tied more closely to the math curriculum.

Gary lives with his wife Barb in the small rural Ontario village of Arthur and spends most summers camping and rockhounding in Australia.

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1787 Agate Street, Eugene, Oregon 97403-1923
Order Desk: 800/336-5191
Phone: 503/346-4414
Fax: 503/346-5890
CompuServe: 70014,2117
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ISBN 1-56484-034-4

Math Activities Using *LogoWriter*

Investigations

Gary Flewelling

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From the Publisher

The purchaser of the *Math Activities Using* LogoWriter—*Investigations* package receives this book, a data disk, and a single-user site license. The *LogoWriter* application is *not* included and *is* necessary for using the disk files.

The single-user site license gives the one person who is designated as the "primary user" of the package the right to make copies of worksheets, disk files, and other material necessary for use in his or her classroom; this right does not transfer to class participants. If the single-user site license is purchased by a school or school district, the intent is that one person be designated as the primary user. If several teachers will use the book, separate packages must be purchased for each individual teacher.

Acknowledgments

For my best friend, my wife Barb.

Thanks also go to Bill Higginson, Queen's University, Kingston, Ontario, for his encouragement; Dan Hawkins, for translating and enhancing an earlier C64 version of this chapter; Doug Benstead, Consultant, Wellington County Board of Education, for helping to "spread the word" at many computer courses and workshops; E. W. Davies for an idea that appeared in *Mathematics in School*, January 1990, and used in Superpoly; and to Linda Kennedy, for desktop publishing.

G.F.

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Preface

A working group of the Canadian Mathematics Educators Study Group (CMESG) met on the campus of the University of Waterloo in June of 1984 to discuss the issue of using Logo in schools. After three days, the group concluded that:

1. Logo, not linked strongly to existing school programs, runs the risk of becoming trivialized.
2. Logo deserves to be integrated into school math programs.
3. Logo is a tool that can be used by teachers to help them implement their mathematics curriculum.
4. Teachers need proof of items 2 and 3.
5. Proof is needed in the form of student activities illustrating items 2 and 3.
6. Samples of activities referred to in item 5 need to be developed and delivered to teachers.

The group agreed that the activities referred to in items 5 and 6 should possess many of the following characteristics.

- can be used by students with a variety of interests and abilities
- allow for a number of lines of inquiry
- can be returned to again and again throughout the school year
- can be used through the grades at varying levels of sophistication
- have links to the mathematics curriculum

- contain an element of choice/freedom/modifiability for both the teacher and the student
- may require the student to construct various procedures to carry out related investigations
- may have to be accompanied by pre-written procedures
- invite the cooperative effort of more than one participant

The conclusions and criteria listed above have guided the author throughout the development of this project.

This series of teacher resource books has been developed to meet two needs. The first, to rescue students from the clutches of computers that drill and control. The second, to supply teachers with computer activities compatible with a mathematics program that emphasizes investigation, problem solving, creativity, hypothesis making, and hypotheses testing.

It is the author's intent that the teacher review and select an appropriate chapter or set of activities within a chapter and integrate them into his or her mathematics program.

Some teachers will want to introduce the whole class to an activity. Others will want to introduce the activity to a smaller group of students who will then act as an expert group for the rest of the class. Still others will simply hand over a copy of the chapter to some students and challenge them to discover something interesting that they can share with others. Some teachers will have a student choose an activity of interest and allow the student to pursue it independently.

In all of these scenarios the teacher will act as a facilitator of the activity, encouraging students to reflect on their work, make and test conjectures, share and discuss findings and problems with other students, ask and investigate "what if" questions, evaluate and refine their own and other students' work, and do some follow-up study. Students can then report their findings in a variety of ways, for example, as a written

report, an oral report to a small group of classmates, a poster or similar display, or a set of conclusions summarized in their journal.

Introduction

This is not a book about Logo; it is a book about math and thinking mathematically. Many chapters require no Logo skills. Where Logo skills are required, they are identified. The basic Logo primitives are described in the appendix at the end of this book. This document does not teach advanced Logo programming skills. See your Logo manual for such details.

Chapter Organization

Each chapter may include as many as seven sections: summary box, getting started, instructions, examples, activities, key summary, and worksheets.

Summary Box

The summary box at the beginning of each chapter includes information about files containing accompanying procedures, appropriate grade levels, color recommendations, and exceptions regarding platform availability. Also included is an illustration of one possible pattern or design.

File Names

Most chapters are accompanied by a group of pre-written *LogoWriter* (TM by LCSI) procedures included in the file named in the summary box.

Grade Level

Many chapters in this book can be used at a variety of levels of sophistication and are identified as multigrade:

P indicates Grades 1-3

J indicates Grades 4-6

I indicates Grades 7-10

S indicates Grades 11-12

Color

Many of the programs used in this book include color. The number of colors available depends on the kind of computer you are using. In general, there are 6 colors on the Apple II, and 3 pen colors for the DOS version; the colors are limited to 15 on the Macintosh.

With a monochrome monitor, the color will alternate between black and white. In general, this will make the turtle alternate between being visible and invisible. If you can't find the turtle, press the key for pen color—usually I = Ink.

Software/Hardware

This book can be used with versions of *LogoWriter* 2.0 or above. Data disks to accompany it are available for the Apple, MS-DOS, or Macintosh platforms. Some programs are not available in all platforms; exceptions are noted in the summary box.

Getting Started

The directions in the Getting Started section of each chapter tell you how to begin using the program for that chapter.

Instructions

Most chapters include an Instructions section. In general, this section describes the details of how the program keys work. Sometimes this section includes an example, particularly when the function of the program is quite complex.

If the chapter is accompanied by pre-written procedures, instructions for using the procedures are given.

Examples

Some chapters have a number of examples that are included to show the teacher or students what the program can

create. Studying these examples will often help them understand how the program functions.

Activities

A series of suggested activities generally accompanies each chapter. Each teacher must judge the appropriateness of any activity for a particular student. Many activities progress better with two students working together.

Teachers are encouraged to take the initiative and modify activities as needed and create new activities as appropriate. They should preview each chapter before giving it to their students.

Key Summary

Most chapters include a key summary at the end. This chart can be copied and kept near the computers where students are working. Details of the key functions are *not* included here.

Worksheets

Some chapters include worksheets. Their purpose is outlined in the appropriate suggested activities.

Tips for Using These Programs

Before You Begin

Make a copy of the disk that came with this book. Store the original in a safe place.

Start *LogoWriter* in the usual manner. If the chapter requires use of a set of pre-written procedures, locate the appropriate file name on the copy of the disk that came with this book. Load the file into *LogoWriter*.

Be sure that the Shapes page that comes with the data disk is also on the disk containing the LogoWriter *files.*

Space

After working with some programs for a length of time, you could get an "out of space" message. If that happens, generally you can start the program again by typing START in the Command Center.

Interrupting a Program

At any time you may interrupt a program that is running by pressing the stop key (Command-. on the Mac; Ctrl-Break for Dos; Apple-S for Apple).

Working With Pictures

Students often will want to save designs that they have made. The following procedure will work for most of the programs in this book. Notes about working with retrieved pictures for particular programs are found in the individual chapters.

To Save a Picture

1. The program should not be running; if necessary, press the Q key or the stop key to exit. You need to be able to enter commands directly in the Command Center.
2. Type SETDISK "A and press Return/Enter.
3. Insert a disk in the A or first drive.
4. Type SAVEPIC "*name of picture*" and press Return/Enter.

To Retrieve a Picture

1. The program should not be running; if necessary, press the Q key or the stop key to exit. You need to be able to enter commands directly in the Command Center.
2. Type SETDISK "A and press Return/Enter.
3. Insert a disk in the A or first drive.
4. Type LOADPIC "*name of picture*" and press Return/Enter.

Printing

Printing the Page

Some of the programs that come with this book include single-keystroke commands for printing the page. If there are no built-in commands for printing, you can exit the program and type

```
PRINTSCREEN
```

to transfer the graphics and text on the page to paper.

Printing Procedures

If you want a copy of the procedures for any particular program, you can flip the page, move the cursor to the Command Center (use the down key), and type

```
PRINTTEXT
```

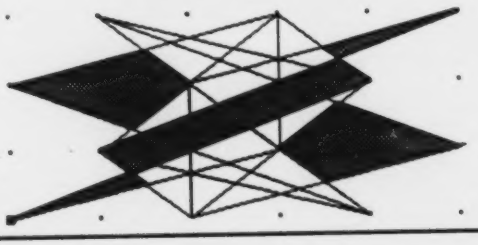
The procedures will appear on paper the same way that they appear on the screen.

Note that the code for all of the files is not documented and in general should not be provided to students.

Chapter 1. De Dot Designer

Filename: DOTCOUNT

Grade Level: J/I
Color: Optional



The DOTCOUNT program allows students to use a computer to create a large variety of geometrical designs by counting and joining dots in an array of dots. These designs can be printed and analyzed for their geometric properties or for their relationship to the numbers involved.

Getting Started

1. Get the page DOTCOUNT.
2. Type START and press Return/Enter.
3. Answer two questions.

   ```
   How many dots in one row?
   How many rows of dots?
   ```

4. After these questions have been answered, a grid of dots is drawn and a > prompt appears. Use the keys

described in the following instructions to connect the dots with lines to create patterns.

Instructions

Moving the Cursor (Arrow Keys)

Use the arrow keys to position the dot-cursor where you want to begin drawing.

Determining Dot-Spacing (N Key)

Press the N key and enter a number to determine the number of dots skipped between endpoints of line segments. This is called the dot-spacing. For more details, see the first example in the following section.

Drawing One Line Segment (Space Bar)

Pressing the space bar causes one line to be drawn. The line drawn is based on the current dot-spacing. When the user starts the program, the dot-spacing is set to 1. The line is drawn from the current location of the dot-cursor to the appropriate "next" dot. For more details, see the first example in the following section.

Note that the dot-spacing can be changed by using the D key described later in this section.

Repeating Lines (R Key)

After pressing the R key, the user enters the number of lines to be drawn. Each line is drawn with the same dot-spacing length, and the beginning of each line is the end of the previous line.

Changing the Dot-Spacing on One Line (D Key)

When the D key is pressed, the user enters a dot-spacing. When a number is entered, one line is drawn. The user is then asked for another dot-spacing. Press the Q key to quit this part of the program.

Changing the Color of the Pen (I Key)

Pressing the I key repeatedly cycles through the available pen colors.

Clearing the Page (K Key)

Pressing the K key removes all of the lines from the grid of dots and then redraws the grid.

Printing the Page (P Key)

Press the P key to print the design on the page.

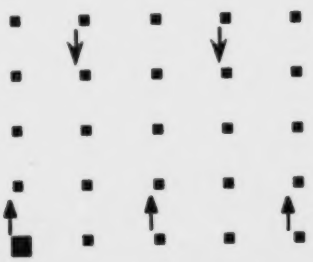
Quitting the DOTCOUNT Program (Q Key)

Press the Q key to quit the program. Type START and press Return/Enter to begin again.

Examples

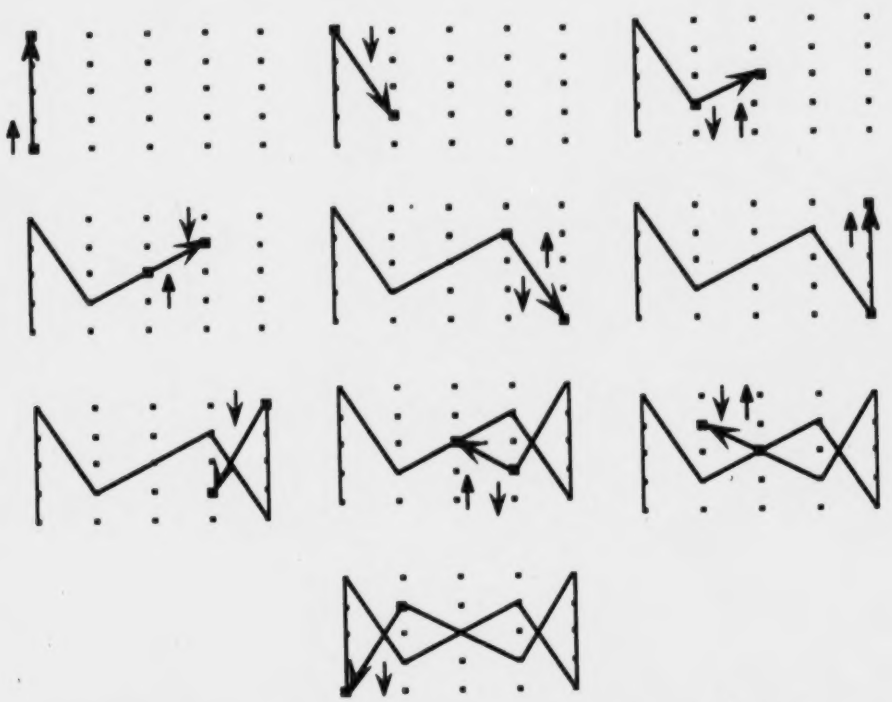
Example 1

Consider an array of 25 dots consisting of five rows of 5 dots. The program counts through the grid of dots as illustrated by the arrows in the following grid. That is, the counting goes up one column of dots, down the next column of dots, up the next, and so forth.



Suppose the dot-spacing is set to 4. The following diagram shows the pattern of lines that would be drawn. Each time the space bar is pressed, the program draws a line segment. The first segment is drawn from dot #1 to the appropriate next dot, which, because the dot-spacing is 4, is dot 5. The next dot after this is dot 9.

Notice what happens after the right-most column of dots is used.

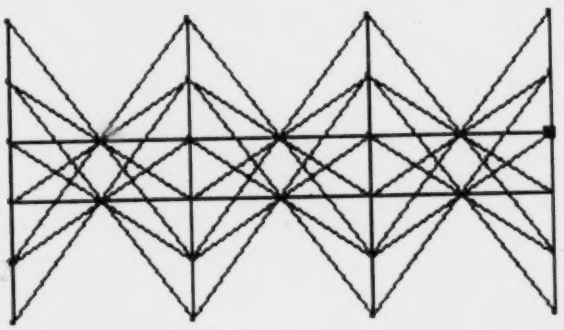


To determine which is the next dot, you can think of counting the number of dots after the current dot or you can think of counting the spaces after the current dot. If you think of counting spaces, you must count the spaces between rows.

Example 2

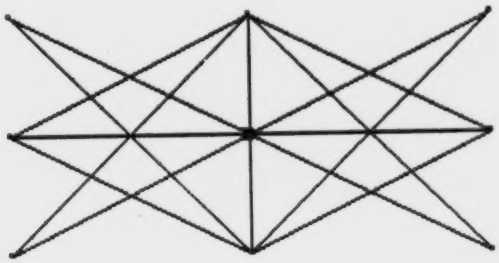
The following drawing shows a 6-by-4 array or grid with the dot-spacing set to 5.

Note that after using the N key to set the dot-spacing to 5, you can use the R key to draw 10 or 20 lines at once.



Example 3

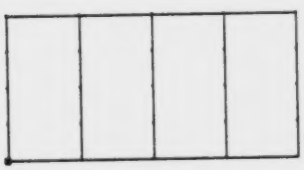
The following drawing was made on a 3-by-3 grid. The D key was used to alternate between a dot-spacing of 3 and 4.



Example 4

The following drawings show a 5-by-5 array with dot-spacing set from 1 to 27.

1.



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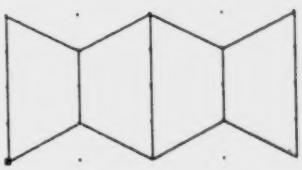

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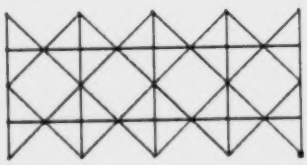

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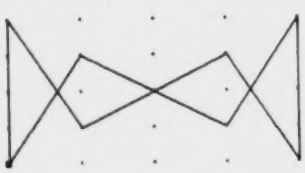
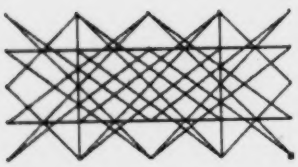


5.

6.

7.



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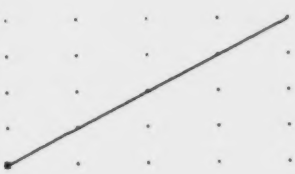

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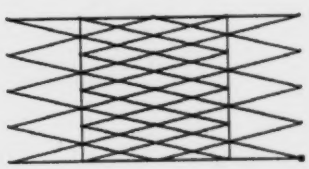

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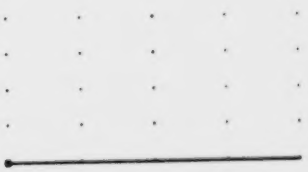

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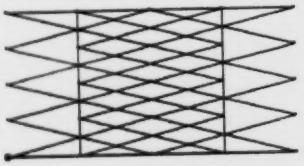

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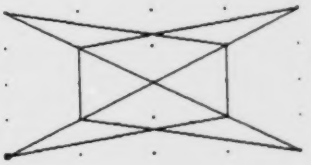

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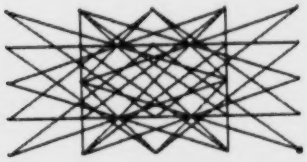

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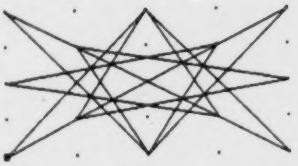

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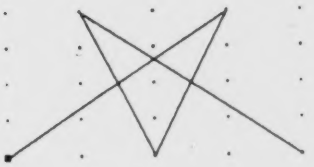

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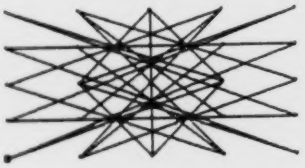

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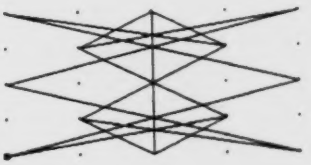

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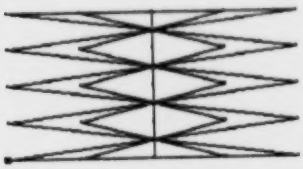

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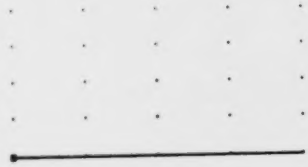

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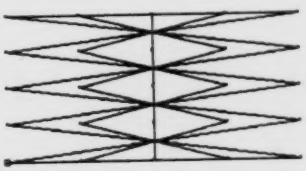

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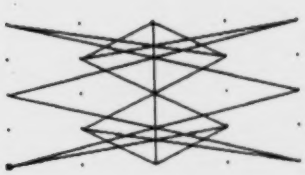

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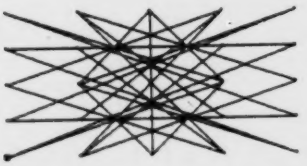

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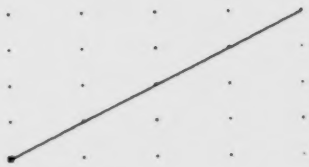

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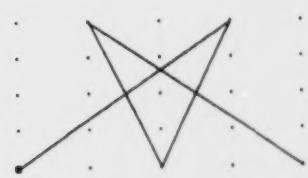

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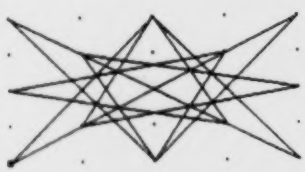

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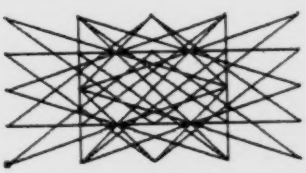
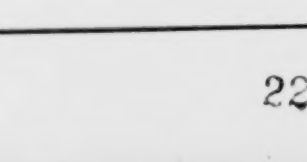




Activities

Activity 1: Free Exploration

Learn to use the DOTCOUNT program to draw different dot designs.

Activity 2: Color Designs

Draw, color, and print one of your designs. When you have completed the design, press the Q key to quit the DOTCOUNT program. Then use Turtle-move and the FILL command to fill areas.

Activity 3: Shape Recognition

Draw and print a design. Color your design either by hand or by using FILL. Make each set of congruent shapes a different color. Write the names of all the shapes in your design under the design.

Activity 4: New Starting Points

Use the same array of dots and the same dot-spacing and investigate the number of different designs you can create by using different starting points. Keep a written account of your findings.

Activity 5: Different Arrays/Equal Dots

A particular number of dots can often be displayed in different ways. For example, twenty-four dots can be displayed as a 2-by-12 array, a 3-by-8 array, a 4-by-6 array, a 6-by-4 array, an 8-by-3 array, and a 12-by-2 array. Using the same dot-spacing and starting point, compare the designs you get making designs in each of these arrays. Share your findings with your classmates.

Activity 6: Predicting the Number of Lines in Designs

Pick small arrays of numbers and investigate the number of lines it requires to complete a design for a particular dot-spacing. Try to predict the number of lines that will complete a design, and then use the R key to enter this number and see if your prediction is accurate.

Activity 7: Using the D Key

Pick a small array of dots, for example, 4 by 6. Draw a design using the D key and counting by 3 dots, then 4 dots, then 3 dots, then 4 dots, and so on, until the design is complete. If you add 3 and 4, you get 7. Investigate other "two-count" designs in which the two numbers also add up to 7. Are there any similarities in the designs you get? Investigate other pairs.

Activity 8: Symmetry

Do all counting designs have symmetry? Investigate and share your finding with others.

Activity 9: Your Own Investigation

Conduct your own dot design investigation. Document your findings, using lots of examples. Share your report with a classmate.

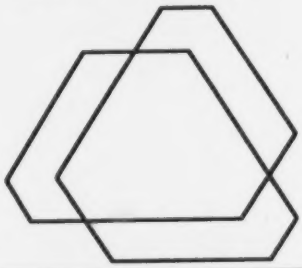
Key Summary for DOTCOUNT

Arrow Keys	Move the dot-cursor to begin drawing a line
N	Determines the dot-spacing
Space Bar	Draws one line using the current dot-spacing
R	Draws the number of lines specified at the current dot-spacing.
D	Changes the dot-spacing and draws one line
I	Cycles through the available pen colors
K	Clears the design
P	Prints the design
Q	Quits the program (Type START and press Return/Enter to begin again.)

Chapter 2. Spirolatera Designs

Filename: SPIROLAT

Grade Level: J/I/S
Color: Optional



The activities in this program are based on the idea of taking a short walk on a path with several "legs." Each leg is followed by turning a fixed angle. A longer walk is created by repeating the short walk over and over again. The path of each long walk results in a network that is rich in geometric ideas. Identification of polygons, congruence, tiling, open and closed curves, parallel and perpendicular lines, symmetry, slides, turns and flips, enlargements, reductions, and similarity are some of the geometry topics that are encountered when studying these networks.

Getting Started

1. Get the page SPIROLAT.
2. Type START and press Return/Enter.
3. Answer two questions.

```
Enter lengths (leave spaces):
Enter turning angle:
```

The lengths of the legs of the path are entered with spaces between them. Thus, entering

```
10 20 30
```

creates a path that has legs that are 10, 20, and 30 turtle steps long, respectively.

The turning angle is the angle to turn between legs of the path. If 90 is entered with the previous path, the path looks like the following drawing.

4. After these two questions have been answered, a > prompt appears and the following keys can be used to create spirolatera.

Instructions

Moving the Turtle (Arrow Keys)

The turtle can be placed anywhere on the page by using the arrow keys. Each time a key is pressed, the turtle moves one turtle-step. (The pen can be up or down. See the following S key instructions.)

Drawing the Path (Space Bar)

Pressing the space bar causes the defined path to be drawn at the current factor. (See the following F key instructions.)

Changing the Turning Angle (A Key)

Pressing the A key allows the user to enter a new turning angle.

Changing the Path (L Key)

When the L key is pressed, the user can enter a new set of lengths for the path.

Changing the Factor (F Key)

Pressing the F key allows the user to enter a multiplier for the path lengths. For example, if the path is

```
10 20 30
```

and the user enters 2 for the factor, the turtle will move 20, 40, and 60 steps the next time the space bar is pressed. Numbers >1 increase the length of the path; numbers <1 (for example, .5) make the path shorter. Entering 1 will return the path to the original size.

Changing the Pen State (S Key)

Repeatedly pressing the S key cycles through the four pen states: Pen Down (PD), Pen Up (PU), Pen Erase (PE), and Pen Reverse (PX). Use PE to erase a path just drawn. Use PU to move the turtle without drawing.

Changing the Color of the Pen (I Key)

Pressing the I key repeatedly cycles through the available pen colors.

Changing the Color of the Background (B Key)

Pressing the B key repeatedly cycles through the available background colors.

Clearing the Page (K Key)

The K key clears the page.

Quitting the SPIROLAT Program (Q Key)

Press the Q key to quit the program. Type START and press Return/Enter to begin again.

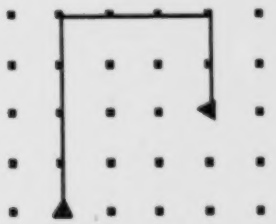
Activities

Activity 1: Constructing Spirolatera On Dot Paper

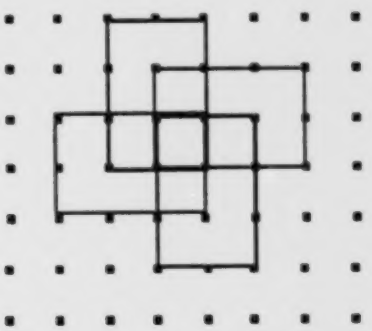
Use square dot paper to make spirolatera designs with 90° turns. Use triangular dot paper to make spirolatera designs with 60° or 120° turns.

For example, start at a selected point and follow this path:

4 steps
turn right
3 steps
turn right
2 steps
turn right

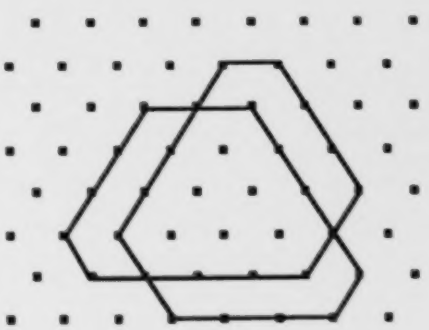


Repeat this path over and over again with the starting point of the next path being the ending point of the previous walk. A spirolatera design results.



Here's a path drawn on triangular dot paper.

1 step
turn 60
3 steps
turn 60
2 steps
turn 60
4 steps
turn 60
repeat 2 times



Construct spirolatera from short walks that consist of two to five legs. Vary the length of one or more of the legs in the short walk.

Investigate the following questions.

1. Which spirolatera close on themselves?
2. Which remain open?
3. Identify equal line segments by drawing each equal set in a different color.
4. Identify families of parallel lines by giving each family a different color.
5. Color each set of identical shapes a specific color.

Activity 2: Writing Spirolatera Procedures

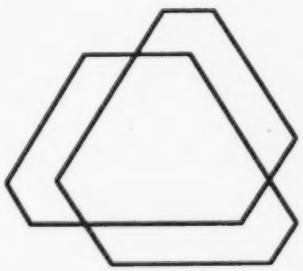
If you have experience writing Logo procedures, write Logo instructions that will draw a sequence of lengths interspersed with turns of constant size. Explore by typing the instructions on one line.

```
FD 20 RT 60 FD 30 RT 60 FD 10 RT 60 FD 40 RT 60
```

Repeat this instruction until the spirolatera design is complete.

The instructions can be copied from the Command Center and put into Logo procedures. In the following example, repeating the procedure entitled IT will generate the design shown.

```
TO IT
FD 20
RT 60
FD 30
RT 60
FD 10
RT 60
FD 40
RT 60
END
```



Activity 3: Investigations

Use the SPIROLAT program to draw a variety of paths. Investigate the following outcomes.

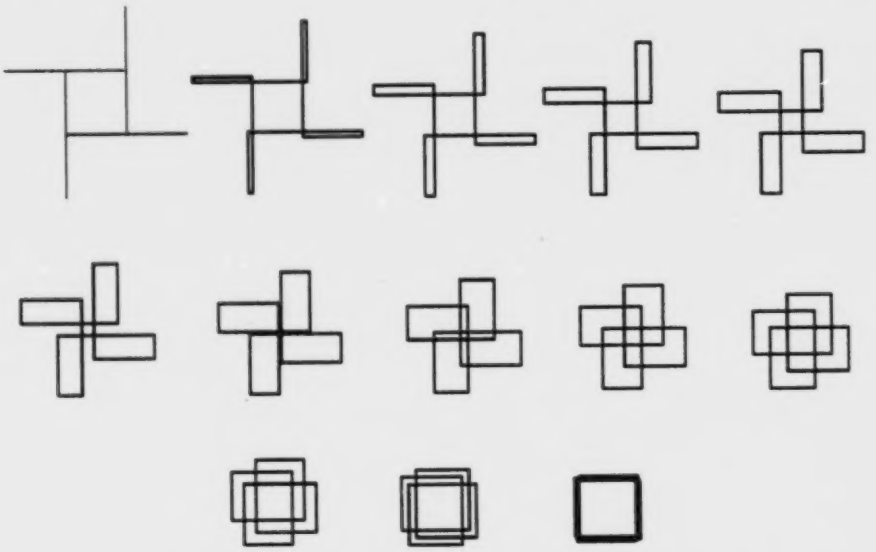
1. The effect of changing the turning angle on the design.
2. The effect of changing the size of one or more of the numbers in the step sequence.
3. The effect of changing the number of numbers in the step sequence.
4. The effect of changing the order of one or more of the numbers in the sequence.
5. The effect of drawing spirolatera designs using PX instead of PD.

Activity 4: Making a Flip Book

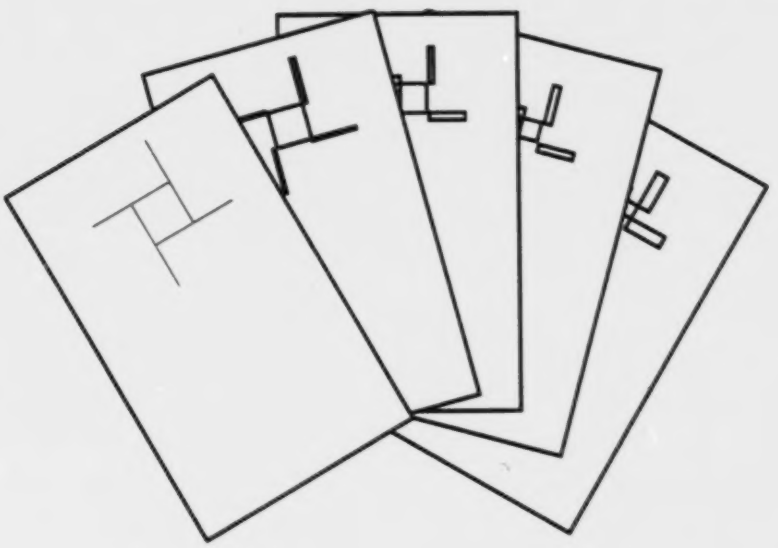
Use the SPIROLAT file to draw a spirolatera design with a turn of 90 and a path of 50 0 25. When the space bar is pressed several times, this design appears.

A systematic change of one or more of the lengths will give you a whole family of related spirolatera.

If 50 0 25 were changed to 48 2 25 and then to 46 4 25 and then to 44 6 25 and so on, you would see something like the following:



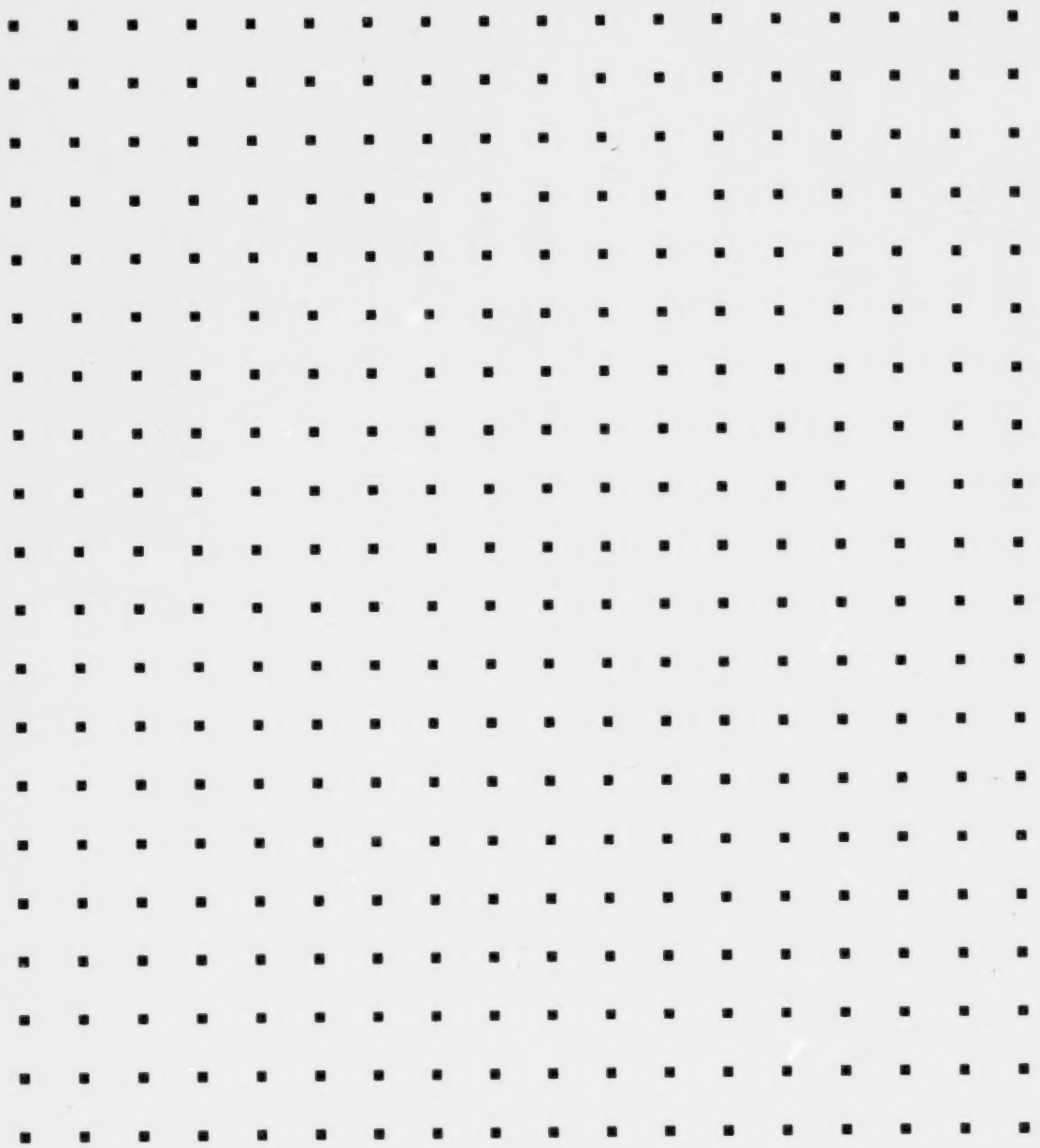
If these spirolatera are then printed onto paper and placed on a sequence of cards, a very interesting geometric animation can be achieved.



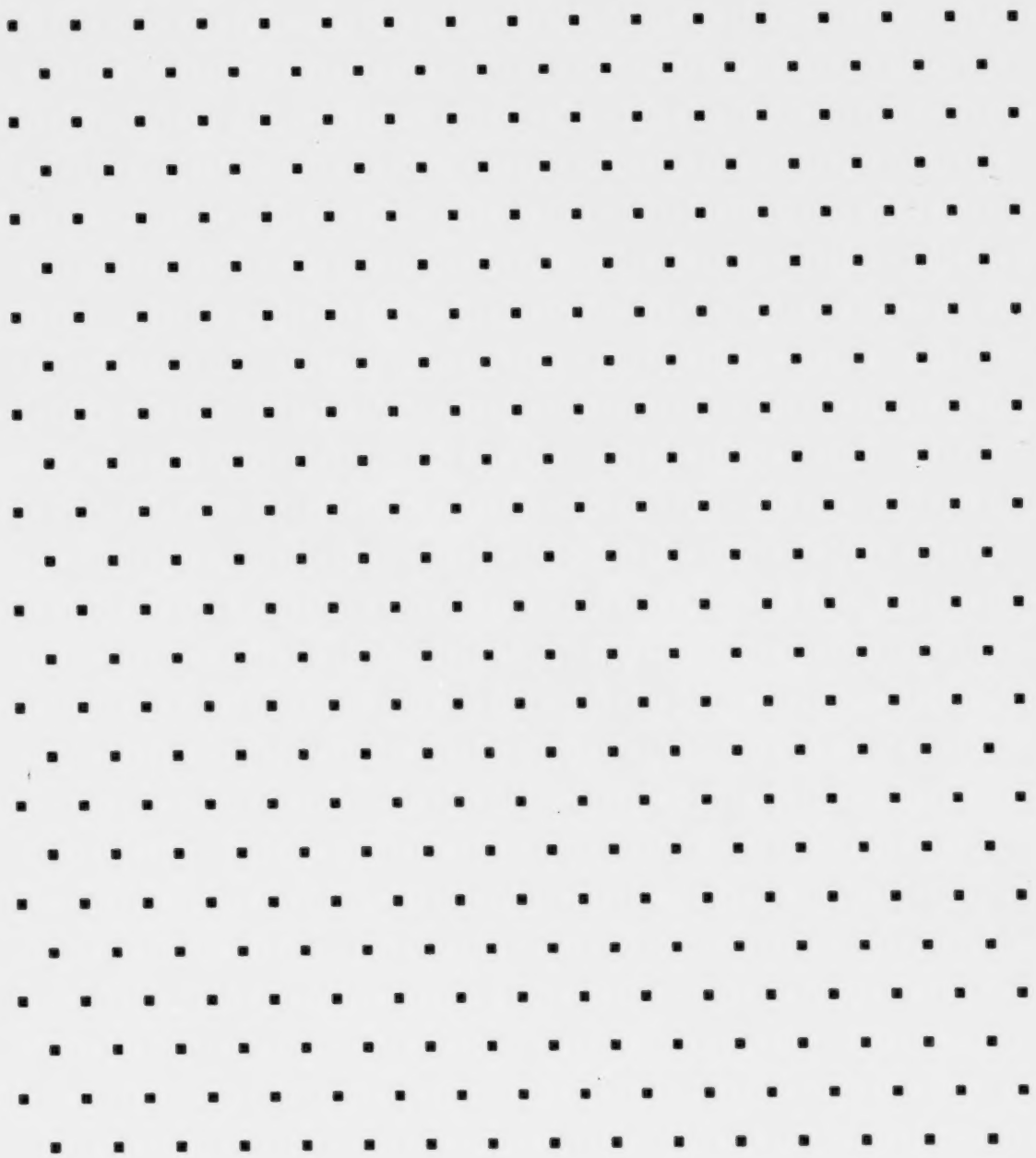
Key Summary for SPIROLAT

Arrow Keys	Move the turtle one turtle-step at a time (Use PU to move without drawing.)
Space Bar	Draws one path using the path definition and turning angle
A	Changes the turn angle between path segments
L	Allows the user to enter a new path definition (Put spaces between path segments.)
F	Changes the factor by which the path lengths are multiplied (Enter 1 to return to the original path size.)
S	Cycles through four pen states: Pen Down (PD), Pen Up (PU), Pen Erase (PE), and Pen Reverse (PX)
I	Cycles through the available pen colors
B	Cycles through the available background colors
K	Clears the design
Q	Quits the program (Type START and press Return/Enter to begin again.)
	To print the graphics and text as they appear on the screen, quit the program, type PRINTSCREEN, and then press Return/Enter

90° Turn Worksheet



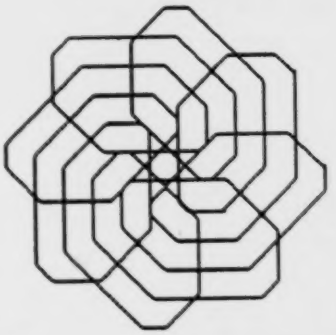
60° and 120° Turn Worksheet



Chapter 3. Digital Root Patterns

Filename: SPIROLAT

Grade Level: J/I/S
Color: Optional



This chapter explores the idea of digital roots.

The digital root of a positive integer is computed by the following process.

- Sum the digits of the number.
- If the sum is >9, repeat the first step.

The single digit answer obtained by this process is the digital root.

For example,

Number	Digital root
5	5
16 → 1 + 6 →	7

85 → 8 + 5 → 13 → 1 + 3 →	4
2457 → 2 + 4 + 5 + 7 → 18 → 1 + 8	9

In this chapter the student works out the digital roots of various numbers and arithmetic expressions. The sequence of numbers can be part of the multiplication or addition table or a complex combination of operations.

Sequences of digital roots can form patterns which become the basis for number of explorations.

Number	Digital root
7 x 1 → 7 →	7
7 x 2 → 14 → 1 + 4 →	5
7 x 3 → 21 → 2 + 1 →	3
7 x 4 → 28 → 2 + 8 → 10→ 1 + 0 →	1
7 x 5 → 35 → 3 + 5 →	8
7 x 6 → 42 → 4 + 2 →	6
7 x 7 → 49 → 4 + 9 → 13 → 1 + 3 →	4
7 x 8 → 56 → 5 + 6 → 11 → 1 + 1 →	2
7 x 9 → 63 → 6 + 3 →	9
7 x 10 → 70 → 7 + 0 →	7
7 x 11 → 77 → 7 + 7 → 14 → 1 + 4 →	5
7 x 12 → 84 → 8 + 4 → 12 → 1 + 2 →	3

Number	Digital root
1 + 5 → 6 →	6
2 + 6 → 8 →	8
3 + 7 → 10 → 1 + 0 →	1
4 + 8 → 12 → 1 + 2 →	3
5 + 9 → 14 → 1 + 4 →	5
6 + 10 → 16 → 1 + 6 →	7
7 + 11 → 18 → 1 + 8 →	9
8 + 12 → 20 → 2 + 0 →	2

$9 + 13 \rightarrow 22 \rightarrow 2 + 2 \rightarrow$	4
$10 + 14 \rightarrow 24 \rightarrow 2 + 4 \rightarrow$	6
$11 + 15 \rightarrow 26 \rightarrow 2 + 6 \rightarrow$	8

Number	Digital root
$1^2 + 2^2 \rightarrow 1 + 4 \rightarrow 5 \rightarrow$	5
$2^2 + 3^2 \rightarrow 4 + 9 \rightarrow 13 \rightarrow 1 + 3 \rightarrow$	4
$3^2 + 4^2 \rightarrow 9 + 16 \rightarrow 25 \rightarrow 2 + 5 \rightarrow$	7
$4^2 + 5^2 \rightarrow 16 + 25 \rightarrow 41 \rightarrow 4 + 1 \rightarrow$	5
$5^2 + 6^2 \rightarrow 25 + 36 \rightarrow 61 \rightarrow 6 + 1 \rightarrow$	7
$6^2 + 7^2 \rightarrow 36 + 49 \rightarrow 85 \rightarrow 8 + 5 \rightarrow 13 \rightarrow 1 + 3 \rightarrow$	4
$7^2 + 8^2 \rightarrow 49 + 64 \rightarrow 113 \rightarrow 1 + 3 + 3 \rightarrow$	5
$8^2 + 9^2 \rightarrow 64 + 81 \rightarrow 145 \rightarrow 1 + 4 + 5 \rightarrow$	1
$9^2 + 10^2 \rightarrow 81 + 100 \rightarrow 181 \rightarrow 1 + 1 + 8 \rightarrow 10 \rightarrow 1 + 0 \rightarrow$	1
$10^2 + 11^2 \rightarrow 100 + 121 \rightarrow 221 \rightarrow 2 + 2 + 1 \rightarrow$	5
$11^2 + 12^2 \rightarrow 121 + 144 \rightarrow 313 \rightarrow 3 + 1 + 3 \rightarrow$	4

Number	Digital root
(1) $(2 + 3) \rightarrow 5 \rightarrow$	5
(2) $(3 + 4) \rightarrow 14 \rightarrow 1 + 4 \rightarrow$	5
(3) $(4 + 5) \rightarrow 27 \rightarrow 2 + 7 \rightarrow$	9
(4) $(5 + 6) \rightarrow 44 \rightarrow 4 + 4 \rightarrow$	8
(5) $(6 + 7) \rightarrow 65 \rightarrow 5 + 5 \rightarrow 11 \rightarrow$	2
(6) $(7 + 8) \rightarrow 90 \rightarrow 9 + 0 \rightarrow$	9
(7) $(8 + 9) \rightarrow 119 \rightarrow 1 + 1 + 9 \rightarrow 11 \rightarrow 1 + 1 \rightarrow$	2
(8) $(9 + 10) \rightarrow 152 \rightarrow 1 + 5 + 2 \rightarrow$	8
(9) $(10 + 11) \rightarrow 189 \rightarrow 1 + 8 + 9 \rightarrow 18 \rightarrow 1 + 8 \rightarrow$	9
(10) $(11 + 12) \rightarrow 230 \rightarrow 2 + 3 + 0 \rightarrow$	5

(11) (12 + 13) → 275 → 2 + 7 + 5 → 14 → 1 + 4 →	5
(12) (13 + 14) → 324 → 3 + 2 + 4 →	9

In the previous examples notice that a patterned sequence of arithmetic expressions will give a repeating sequence of digital roots. For example, the sequence of expressions 7x1, 7x2, 7x3, ··· gives the following repeating set of digital roots: 7 5 3 1 8 6 4 2 9.

Each patterned sequence of expressions can be used to generate a spirolatera design.

Getting Started

Get the page SPIROLAT using the instructions from the previous chapter.

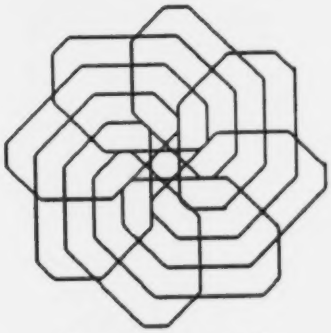
Because digital roots are numbers from 1 through 9, it is best to multiply them by some factor, such as 5, before entering them so that the movement of the turtle will be visible.

Examples

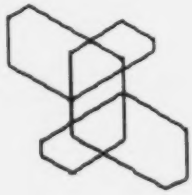
Example 1

Numbers	7x1	7x2	7x3	7x4	7x5	7x6	7x7	7x8	7x9
Digital Roots	7	5	3	1	8	6	4	2	9
Steps	35	25	15	5	40	30	20	10	45

Enter these steps as the lengths of the legs and 45° or 60° as the turning angle; the following designs will result.



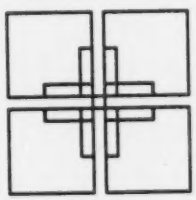
45° turning angle



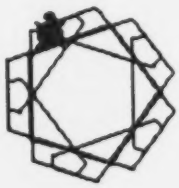
60° turning angle

Example 2

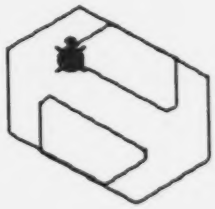
Numbers	1x1	2x2	3x3	4x4	5x5	6x6	7x7	8x8	9x9
Digital Roots	1	4	9	7	7	9	4	1	9
Steps	5	20	45	35	35	45	20	5	45



90° turning angle



72° turning angle



60° turning angle

Example 3

Numbers	Digital Roots	Steps
$1^2 + 2^2$	5	25
$2^2 + 3^2$	4	20
$3^2 + 4^2$	7	35
$4^2 + 5^2$	5	25
$5^2 + 6^2$	7	35

$6^2 + 7^2$	4	20
$7^2 + 8^2$	5	25
$8^2 + 9^2$	1	5
$9^2 + 10^2$	1	5

90° turning angle

Activities

Activity 1: Spirolatera Designs From Given Arithmetic Expressions

Using the sequences of number expressions provided, work out their digital roots. Create a spirolateral design with the repeating digital roots.

Activity 2: Student Created Number Patterns

Create your own sequences of number expressions. Proceed as in Activity 1. Number patterns that generate interesting designs can be written on cards for other students to use. Be sure to include the turning angle.

For example, you could describe a number sequence by writing

```
1x7 2x7 3x7 ... with a turning angle of 45°
```

on a card.

Activity 3: Exploration

There are many avenues of exploration of digital root patterns—limited only by the curiosity and insight of the investigator. Follow your own avenues of inquiry.

You might investigate such questions as

1. Do all multiplication sequences result in a repeating set of digital roots?
2. Do all repeating sets of digital roots repeat after the same number of numbers?
3. Can digital root patterns be predicted?
4. Can different sets of numbers have the same sequence of digital roots?
5. What will some uniform change in a number expression sequence do to the resulting spirolatera design?

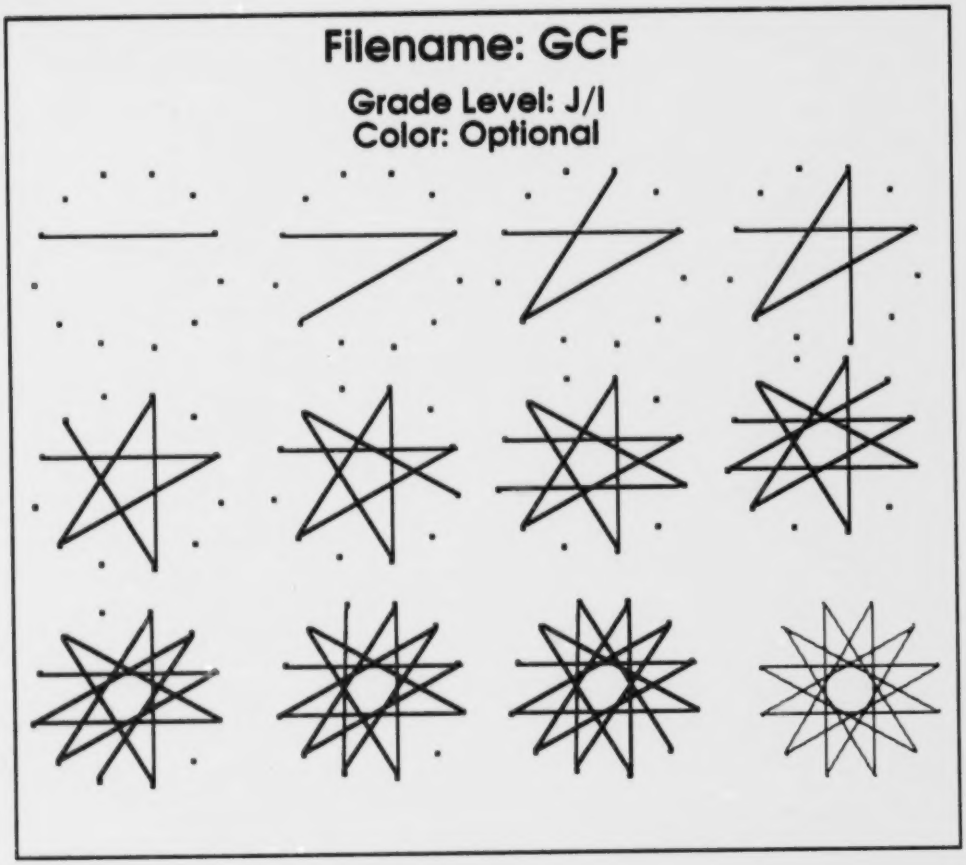
Activity 4: Using Spirolatera Designs

Print interesting spirolatera designs resulting from previous activities. Use these printed designs in further activities.

For example, a pattern could be duplicated, colored, cut out, and reassembled with glue into a polyhedral soccer ball.

The 60° design in Example 2 in this chapter could be duplicated, cut out, and pasted into an interesting piece of art.

Chapter 4. GCF and Line Designs



The GCF (Greatest Common Factor) program lets the user place any number of points, evenly spaced, around a circle. A design is then created by joining points in order around the circle. The design produced depends on the number of points skipped before joining one point on the circle to a second point on the circle.

The summary box design was created using a 12-point circle on which 4 points were skipped and the fifth was joined with the first.

The challenge is in predicting the design that will result in joining every *k*th point on a circle containing *n* points.

Getting Started

1. Get the page GCF.
2. Type START and press Return/Enter.
3. Answer a series of questions. The first question asks the user about clearing the page.

   ```
   Clear the screen (Y/N)
   (Press Q to stop.)
   ```

   Pressing the Y key (yes) causes all of the graphics on the screen to be removed. Pressing the Q key quits the GCF program.

   Note that three different drawings can be placed on the screen at a time.

   The next question asks if the user wants the drawing labeled with the number of points drawn and the number of points skipped.

   ```
   Labeling on? (Y/N)
   ```

   Answering Y (yes) causes a label to be added to the diagram after it is drawn.

   The third question asks the user to enter the number of points to be drawn around the circumference of a circle.

   ```
   How many points on the circle?
   ```

   The final question asks the user to indicate how many points will be *skipped* when lines are drawn between points.

   ```
   Skip how many points?
   ```

   Note that entering 6 means that six points will be skipped and every seventh point will be connected.

4. Type PRINTSCREEN and press Return/Enter (after quitting the program) to print the page.

Activities

Activity 1: Off-Computer Designs

With a straight edge and a number of equally spaced points marked on a circle, construct a variety of line designs using the procedure described previously. A worksheet with an 11-point circle is included at the end of this chapter.

Activity 2: On-Computer Designs

Repeat Activity 1 using the GCF program.

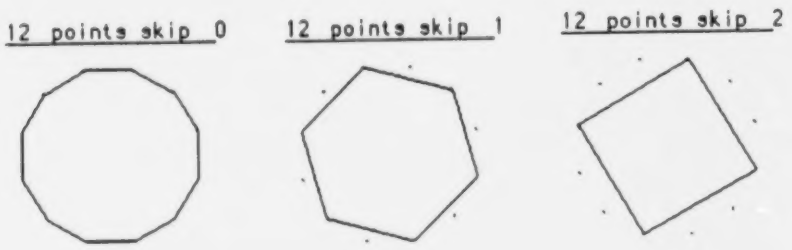
Activity 3: Two-Person Competition

You and a partner can have a friendly competition using this program. Draw a design on one part of the screen. Challenge your partner to draw another identical design skipping a different number of points. If it is not possible to do so, your partner should say so. Continue drawing identical designs until all possibilities have been exhausted. Keep score and alternate roles.

Notice in the following activity that a 12-point circle with 7 points skipped is the same as a 12-point circle with 3 points skipped.

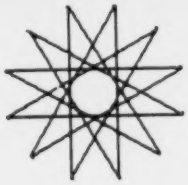
Activity 4: GCF

What does GCF, standing for the greatest common factor, have to do with the following line designs?



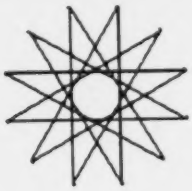
12 points skip 3

12 points skip 4



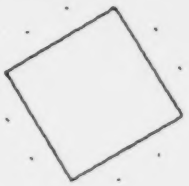
12 points skip 5

12 points skip 6

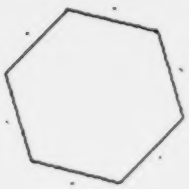


12 points skip 7

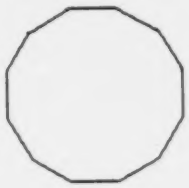
12 points skip 8



12 points skip 9



12 points skip 10



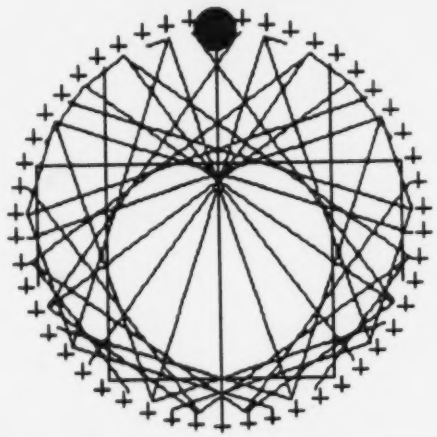
12 points skip 26

11-Point Circle Worksheet

Chapter 5. Two Frogs and a Frisbee

Filename: FROGS

Grade Level: J/I
Color: Optional



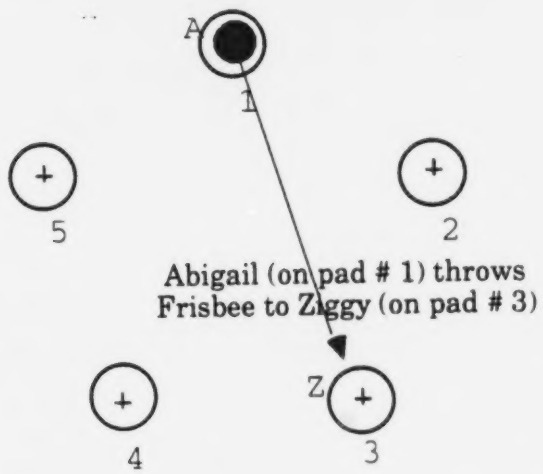
In this program the student is introduced to two frogs, Abigail and Ziggy. Each morning these two frogs head into the pond for a game of toss the frisbee. The frogs hop around the pond from lily pad to lily pad tossing the frisbee back and forth to each other. The lily pads are spread out evenly around the edge of the pond. The path of each toss of the frisbee is displayed on the screen. The student can control the number of lily pads on which Abigail hops, the number of lily pads on which Ziggy hops, the length each frog jumps, the two lily pads from which the frogs start, and the clockwise or counterclockwise direction in which the two frogs hop. Many investigations in number theory and geometry can be conducted with this program.

Getting Started

1. Get the page FROGS.
2. Type START, then press Return/Enter.
3. Answer a series of questions.

```
How many lily pads for Abigail?
How many lily pads long is her jump?
How many lily pads for Ziggy?
How many lily pads long is his jump?
```

A circle is drawn showing the lily pads for each frog. Abigail's lily pads are represented by small vertical lines; Ziggy's lily pads are represented by small horizontal lines. The frisbee, Abigail, and Ziggy are all initially placed on the first lily pad, at the top of the circle.



The Lilypond

Abigail is represented by a letter A and Ziggy by a letter Z. The frisbee is a filled circle. When the lilypond is first drawn, Abigail, Ziggy, and the frisbee are all on the first lily pad and you can see only the frisbee. After the first throw—from Abigail to Ziggy—Ziggy jumps to his "next"

lily pad. Then Ziggy throws the frisbee to Abigail, and Abigail jumps to her next lily pad.

Note that at any point, only one frog-letter will be visible because the other frog-letter is hidden by the frisbee it is "holding."

4. Explore a variety of patterns with the following keys.

Instructions

Talking to Frogs/Frog Status (A and Z Keys)

If you wish to give instructions to Abigail, you press the A key and then use the instruction key. If you wish to give instructions to Ziggy, you use the Z key. When you press the A or Z key, you also see the status of that particular frog. For example

```
Abigail Frog
Number of pads 5
Jump of 1 clockwise
On pad 1
Number of tosses 0
```

Use the A and Z key at any time to see the status of either frog.

Throwing the Frisbee (F Key)

Press the F key to throw the frisbee once.

Note the toss alternates between the two frogs. After a toss is completed, the frog that tossed the frisbee jumps to the next pad.

Repeating Throws (R Key)

When the R key is pressed, the user is prompted to enter the number of times to throw the frisbee.

Moving a Frog (M Key)

When working with the FROGS program, only one frog is active at a time, depending on whether you last pressed the A key or the Z key. The M key is used to move the active frog to a particular lily pad.

Changing the Jump Length (J Key)

Pressing the J key allows the user to change the length of the jump for the active frog.

Changing the Direction of a Jump (D Key)

Pressing the D key allows the user to switch between clockwise and counterclockwise movement for the active frog.

Clearing the Frog Status (C Key)

Pressing the C key removes the text describing the active frog from the page.

Clearing the Page (K Key)

Using the K key removes all of the lines from the grid of dots and then redraws the grid.

Changing the Color of the Frisbee Path (I Key)

Pressing the I key repeatedly cycles through the available colors used to draw the paths between lily pads.

Printing the Page (P Key)

Use the P key to print the page. When the P key is pressed, a message is displayed asking if you want to print. Press the Y key (yes) to print the page.

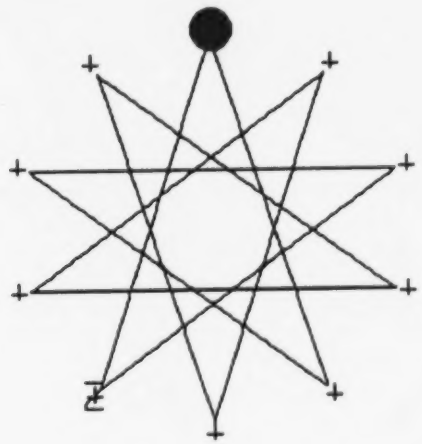
Quitting the FROGS Program (Q Key)

Press the Q key to quit the program. Type START and press Return/Enter to begin again.

Examples

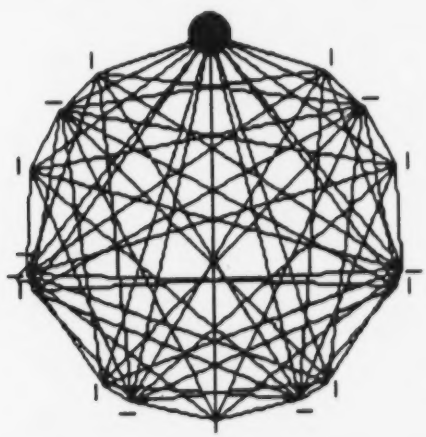
Example 1

Frog	Number of lily pads	Jump length	Starting pad number	Direction
Abigail	10	1	1	Clockwise
Ziggy	10	1	7	Clockwise



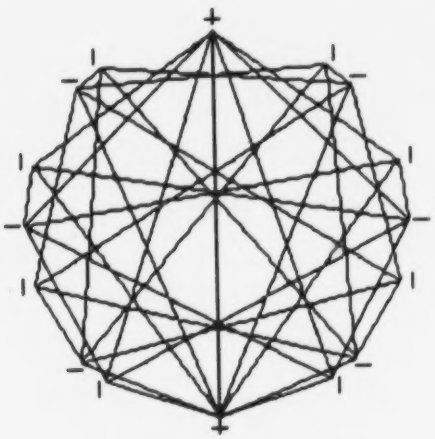
Example 2

Frog	Number of lily pads	Jump length	Starting pad number	Direction
Abigail	10	1	1	Clockwise
Ziggy	7	1	1	Clockwise



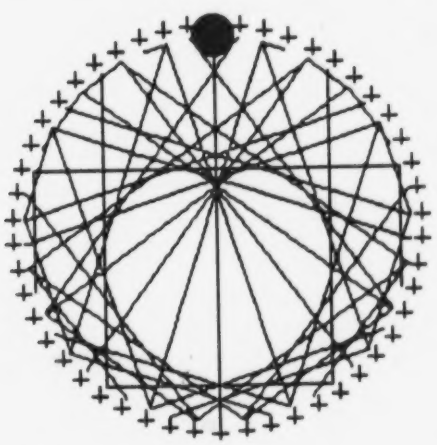
Example 3

Frog	Number of lily pads	Jump length	Starting pad number	Direction
Abigail	10	1	1	Clockwise
Ziggy	8	1	1	Clockwise



Example 4

Frog	Number of lily pads	Jump length	Starting pad number	Direction
Abigail	50	2	1	Clockwise
Ziggy	50	3	1	Clockwise



Activities

Activity 1: Free Exploration

Spend some time learning to use the program by creating a number of frisbee path designs.

Activity 2: Different Starting Points

Investigate a variety of frisbee paths in which both frogs use the same number of pads and have the same jumping length. Change only the starting point of Ziggy. Try to predict the results involving a small number of lily pads.

Activity 3: Jumping in Different Directions

Investigate a variety of frisbee games in which Abigail and Ziggy jump in opposite directions. Start with simple cases first involving small numbers of lily pads.

Activity 4: Different Jumping Lengths

Investigate a variety of frisbee paths in which the only difference is the length of jumps of the two frogs.

Activity 5: Different Numbers of Lily Pads

Investigate a variety of frisbee paths in which the only difference is the number of lily pads on which the frogs jump.

Activity 6: Number of Tosses In a Complete Design

Predict the number of paths needed to complete a design. Use a small number of lily pads to investigate a variety of simple paths.

Key Summary for FROGS

A	Makes Abigail active and shows her status
Z	Makes Ziggy active and shows his status
F	Throws the frisbee one time
R	Repeats the frisbee throw
M	Moves the active frog to a lily pad you specify
J	Changes the length of the jump for the active frog
D	Changes the direction of the active frog
C	Removes the frog status from the page
K	Clears the graphic and redraws the circle
I	Cycles through the available colors for the path
P	Prints the design
Q	Quits the program (Type START and press Return/Enter to begin again.)

Chapter 6. Repeating Decimals

Filename: REPEAT

Grade Level: I/S
Color: Optional

This program allows users to input values for the numerator and denominator of a rational number and see the resulting set of repeating decimals corresponding to that number. In addition, this set of digits can then be used to generate spirolatera designs. (See Chapter 2.) The user can thus investigate numeric and geometric properties of rational numbers.

All fractions can be expressed as decimals through a process of division. Only decimal representations of proper fractions need to be examined, because mixed or improper fractions can be expressed as a whole number plus a proper fraction.

Remainders play a critical role in determining the character of a fraction's decimal representation.

If the final remainder is zero, as in the following example, we say the decimal terminates.

```
  0.125
8) 1.000
    8
    20        1/8 = 0.125 (terminating)
    16
     40
     40
      0
```

If one of the remainders associated with a partial quotient reappears, the decimal will repeat.

```
  0.142857
7) 1.000000
   7
    30
    28
     20
     14
      60      1/7 = 0.142857142857142857...
      56
       40
       35
        50
        49
         1
```

This is often written as either

$1/7 = 0.\dot{1}42857\dot{}$

or

$1/7 = 0.\overline{142857}$

In the previous example, the partial quotients 1, 4, 2, 8, 5, 7 arise from the remainders, 1, 3, 2, 6, 4, 5. The next remainder, 1, gives a partial quotient of 1 and the partial quotients 1,4,2,8,5,7 then appear again and again.

There can't be more than n - 1 numbers in the repeating part of the decimal if the divisor of the fraction is n.

Getting Started

1. Get the page REPEAT.
2. Type START and press Return/Enter.
3. Enter the numerator and denominator of a fraction. The computer then produces the decimal. For example

```
The fraction 3 / 7 equals 0.4 2 8 5 7 1
With the 6 digits 4 2 8 5 7 1 repeating
```

The digits which make up this decimal can then be used to generate spirolateral designs. (See Chapter 2 for a discussion of spirolaterals.)

Instructions

After the user enters the numerator and denominator of a fraction and the computer produces the decimal, the program then asks if the user wants a graphical representation of the decimal. If the answer is yes, a design appears on the page. (If the answer is no, the program stops. Type START and press Return/Enter to begin again.)

The turtle continuously draws a pattern using 10 as the length and 90 as the angle of turn. In the Command Center you see

```
Press P to pause, L for length change, A for
   angle change.
```

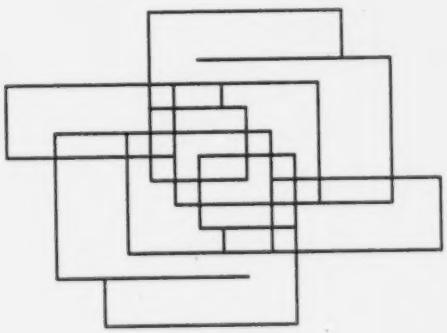
If the P key is pressed, the turtle stops drawing. The user can then stop the program by pressing the Q key or return to watching the turtle draw by pressing the C key to continue.

If the A key is pressed, the user is asked to enter an angle of turn. If the L key is pressed, the user is asked for the length of each leg of the spirolateral. The page is cleared after each angle or length change.

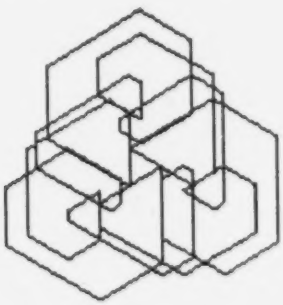
Examples

Example 1

The following spirolateral results from the fraction 1/23 with a length of 2 and an angle of 90°

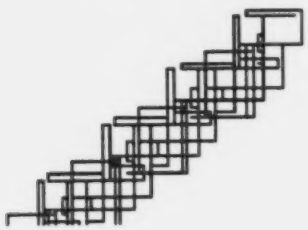


while the following spirolateral was generated using 1/23 with a length of 1 and an angle of 60°.



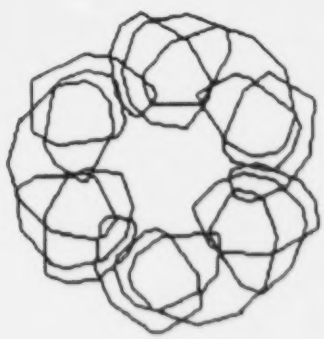
Example 2

This pattern was generated using 1/61, line segment length .5, and turning angle 90°. The pattern repeats in such a way that it wraps around the page.



Example 3

This pattern was generated using the same parameters as in Example 2, except with a 32° turning angle.



Activities

Activity 1: Repeating Decimals

Investigate patterns in repeating decimals by hand, using long division and simple proper fractions. For simple fractions, use a hand-held calculator.

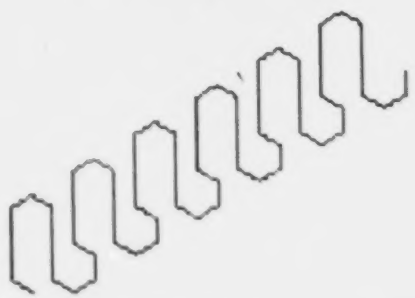
Activity 2: Repeating Decimals

Use the REPEAT page to continue the investigations you started in Activity 1.

Chapter 7. Super Wriggly Worms

Filename: WORMY

Grade Level: J/I
Color: Optional



WORMY lets the user create a worm of specified length, make it come up out of its hole, and make it wriggle around the page. The user can define the way in which the worm wriggles and make the worm create a large variety of geometric patterns by making the worm repeat its wriggly motion again and again over the page. The user can conduct a variety of investigations of area and of two-dimensional shapes.

Getting Started

1. Get the page WORMY.
2. Type START and press Return/Enter.
3. Enter a whole number worm length.

When defining a worm length keep in mind that the initial worm length is 10 turtle steps.

4. Create patterns on the page with the following keys

   Note that the program must be restarted to change the worm length.

Instructions

Defining a Worm (Arrow Keys and A Key)

The arrow keys and the A key are used to define the shape of the worm.

The up arrow key (↑) moves the worm one unit forward in the direction that the worm is heading.

The left arrow key (←) turns the worm to the left and draws one worm segment.

The right arrow key (→) turns the worm to the right and draws one worm segment.

Pressing the A key allows the user to change the angle of turn at any point in the definition of a worm. The angle of turn is initially set to 90° but can be changed at any point during the definition of a worm.

Note that the A key is used while defining a worm. Pressing the A key after a worm is defined does not change the shape of the current worm.

You must *enter the* same *number of moves and turns as you specified for worm length. A number less than the length entered may result in an error message.*

If you use the arrow keys after *you have entered the number of moves equal to the length of the worm you specified, only the last moves will be recorded. That is, if you specified a five-move worm and press an arrow key again, the last four moves from the original worm plus the last press of the arrow key now define the worm.*

Making the Worm Wriggle (W Key)

Pressing the W key makes the worm wriggle in such a way that it's path is recreated in the "next" location on the page. The worm leaves a path if the T key is pressed to cause it to leave a trail.

Leaving a Trail (T Key)

When the WORMY program begins, the message "Worm trail off" appears in the upper left corner of the page. Pressing the T key toggles between "Worm trail on" and "Worm trail off." If color is available, the worm leaves a colored path.

Clearing the Page (K Key)

Pressing the K key removes all worm trails. The definition of the worm remains the same.

Changing the Worm Size (S Key)

Pressing the S key allows the user to enter a number to change the size of the worm. If a number >1 is entered, the worm draws its next path larger than the original; if a number <1 is entered, the worm draws its next path smaller.

Thus, entering 2 for the worm size makes the worm twice as big. Entering 1 makes the worm its original size.

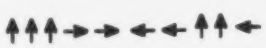
Quitting the WORMY program (Q Key)

Pressing the Q key quits the WORMY program. Type START and press Return/Enter to begin again.

Examples

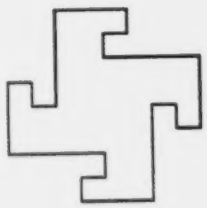
Example 1

Using the following keystrokes creates a worm of length 10.



Example 2

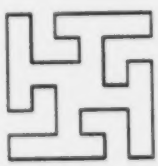
First, use the T key to turn the worm trail on. Then use the W key several times with the worm from Example 1.



Example 3

The following worm has a length of 12 and uses 90° as its turn angle.

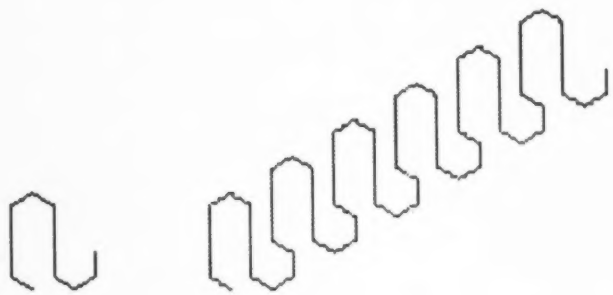
Pressing the W key several times gives



Example 4

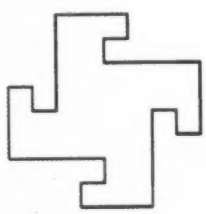
The following example is a worm of length 12 with 60° turns.

Note that the first step in drawing the worm is a left turn.

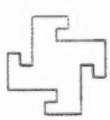


Example 5

Use the S key to change the size of the drawing. If your original drawing looks like this



then entering 0.5 for the size gives this.



Activities

Activity 1: Exploration

Spend some time creating worms. Be sure you understand the difference between pressing the left and right arrow keys to turn the worm and pressing the up arrow key to move forward.

Activity 2: Wriggly Patterns

Create a variety of worms. Then use the T key to make the worm leave a trail on the screen. Press the W key a number of times to investigate the trails that different wriggle patterns make.

Activity 3: Investigating Worms With Different Turns

Use the same worm several times, but change the turning angle. Be sure to carefully record your worm path so that you can reenter the same keys after you change the angle. Use the W key and study the trail the worm leaves each time.

Activity 4: Worms with Varied Turns

As you define your worm, you can change the angle of turn. For example, you can press the left arrow key to turn left 90° then press the A key and change the angle to 60°. Then press the left arrow key again and the worm turns 60°. Explore creating worms with a variety of angles. Perhaps alternate two angles in a single worm. Then reverse the order in which you use the angles. Explore patterns that emerge.

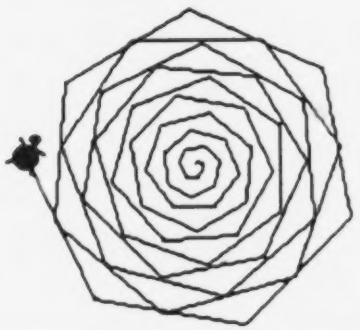
Key Summary for WORMY

Up Arrow	Defines one worm segment in the direction the worm is facing and draws one worm segment
Left arrow	Defines a worm segment that turns the worm to the left by the defined angle and draws one worm segment
Right arrow	Defines a worm segment that turns the worm to the right by the defined angle and draws one worm segment
A	Changes the turn angle for the next worm segment
W	Causes the current worm definition to repeat once
T	Toggles between leaving a worm trail and not leaving a worm trail
K	Clears the worm path
S	Changes the size of the worm
Q	Quits the program (Type START and press Return/Enter to begin again.)
	To print the graphics and text as they appear on the screen, quit the program, type PRINTSCREEN, and then press Return/Enter

Chapter 8. Superpoly

Filename: SUPERPOL

Grade Level: I/S
Color: Optional



The instruction

`FD` *`length`* `RT` *`length`*

is one of the best known Logo drawing instructions. The SUPERPOL program allows the user to investigate often surprising effects of repeating the instruction and changing the length and angle in this instruction.

In this program, the user can set

- an initial length
- a value by which the length can change
- an initial angle
- a value by which the angle can change

For more advanced users, the program also allows entering

- a function of the length to define magnitude of the length moved forward
- a function of the angle to define the magnitude of the right turn

Getting Started

1. Get the page SUPERPOL.
2. Type START and press Return/Enter.
3. Enter a series of values.

```
Enter length:
Enter length change:
Enter turning angle:
Enter change in angle:
```

4. After values have been entered, a > prompt appears and the following keys can be used.

Instructions

Running One Instruction (G Key)

Press the G (Go) key to run the

```
FD length RT length
```

one time. Each time the G key is pressed, the command is run again with the change in length and angle added each time.

Running the Instruction Multiple Times (N Key)

When the N key is pressed, the user enters the number of times the instruction

```
FD length RT length
```

is to be run.

Note that the change in length and angle is added each time the instruction is run.

Changing the Values (R Key)

Pressing the R key allows the user to reset the four initial values. The current values of the length, change in length, the angle, and change in angle are displayed. New values can be entered for each of the four values. Pressing Return/Enter leaves a value unchanged.

Clearing the Page (K Key)

Use the K key to remove all of the lines on the page.

Changing the Pen State (S Key)

Pressing the S key repeatedly toggles between Pen Up (PU) and Pen Down (PD). The messages "Trail shown" and "No trail shown" appear on the page.

Changing the Color of the Pen (I Key)

Pressing the I key repeatedly cycles through the available pen colors.

Changing the Size Factor (F Key)

Pressing the F key allows the user to enter a number to change the size of the drawing. If a number >1 is entered, the design is drawn larger than the original; if a number <1 is entered, the design is drawn smaller. Enter 1 to return to the original design size.

Note that entering 1 will not cause the diagram to begin again. You must use the R key to reset the values to the original.

Defining New Functions (D Key)

The D key is used to define new length and angle functions. The length functions must be defined in terms of the variable LEN and the angle functions must be defined in terms of ANG. The operators that may be used are

+	-	/	*	int	
random	remainder	sqrt	arctan	cos	sin

Be sure to leave spaces between operators, constants, and variables. Be sure to precede variable names with ":" and do not *enter either FD or RT. (See Example 5.)*

Quitting the SUPERPOLY Program (Q Key)

Press the Q key to quit the program. Type START and press Return/Enter to begin again.

Examples

Example 1

Draw a fixed length and fixed angle pentagon:

```
Enter length: 20
Enter length change: 0
Enter turning angle: 72
Enter change in angle: 0
```

Example 2

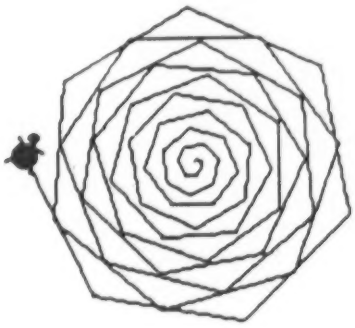
To draw a fixed length, fixed angle star polygon:

```
Enter length: 40
Enter length change: 0
Enter turning angle: 100
Enter change in angle: 0
```

Example 3

To draw a changing length, fixed angle spiral:

```
Enter length: 0
Enter length change: 1
Enter turning angle: 55
Enter change in angle: 0
```



Example 4

To draw a fixed length, changing angle pattern

```
Enter length: 1
Enter length change: 0
Enter turning angle: 0
Enter change in angle: 5
```

The scale factor was 3.

Example 5

To create a pattern with a new function for the length.

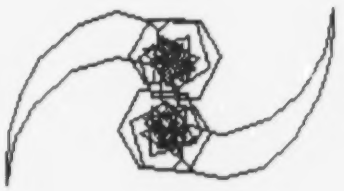
```
Enter length: 0
Enter length change: 20
Enter turning angle: 0
Enter change in angle: 5
```

and then use the D key to enter a function.

```
Enter length function: 20 * SIN :LEN
```

and then press Return/Enter for the angle function, leaving it

```
RT :ANG
```



Activities

Activity 1: Fixed Lengths and Fixed Angles

Investigate the kinds of designs that can be drawn by keeping the length and angle fixed values, that is, entering 0 for "change in length" and "change in angle" prompts.

For which fixed angles are regular polygons drawn? (See Example 1.)

For angles other than those that draw regular polygons, how many sides will the corresponding star polygon have? (See Example 2.)

Activity 2: Fixed Angles

Investigate the spirals resulting from keeping the angle fixed in value while defining changes in the length moved. (See Example 3.)

Activity 3: Fixed Lengths

Investigate designs that result from keeping the length fixed and defining changes in the turning angle. (See Example 4.)

Activity 4: Defining Length Functions

Investigate designs resulting from different length functions.

Activity 5: Defining Angle Functions

Investigate designs resulting from different angle functions.

Activity 6: Define Length and Angle Functions

Investigate the designs that result from different length and angle functions.

Key Summary for SUPERPOL

G	Runs an instruction one time
N	Runs an instruction multiple times
R	Change the value of the length, angle, change in length, and change in angle
K	Clears the page
S	Cycles between Pen Up (PU) and Pen Down (PD)
I	Cycles through the available pen colors
F	Changes the size factor for the design
D	Defines new length and angle functions
Q	Quits the program (Type START and press Return/Enter to begin again.)
	To print the graphics and text as they appear on the screen, quit the program, type PRINTSCREEN, and then press Return/Enter

Appendix A
Summary of *LogoWriter* Keys for the Apple IIe/IIc

General Purpose Keys

Flip	Apple-F
Up	Apple-U
Down	Apple-D
Quit, leave page	Esc
Stop	Apple-S
Help	Apple-0

Word-Processing Keys

Select	Apple-1
Cut	Apple-2
Copy	Apple-3
Paste	Apple-4
Erase to End of Line	Apple-6
Next Screen	Apple —>
Previous Screen	Apple <—
Top of Page	Apple-Up Arrow
Bottom of Page	Apple-Down Arrow
Beginning of Line	Apple-B
End of Line	Apple-E
Open a Line	Apple-O
Delete Right	Control-D

Shapes Page Keys

Quit, leave page	Esc
Flip	Apple-F

On the flip side of the page only:

Next Screen (Next Shape)	Apple —>
Previous Screen (Previous Shape)	Apple <—
Up, Down, Left, Right	Arrow keys
Cut	Apple-2
Copy	Apple-3
Paste	Apple-4
Make or Erase Block	Space Bar

Graphics Keys

Turtle-Move	Apple-9
Label	Apple-8
Leave Label or Turtle-Move	Esc

Contents Keys

Top of Page	Apple-Up Arrow
Bottom of Page	Apple-Down Arrow
Remove a Page (Erase to End of Line)	Apple-6

Summary of the *LogoWriter* Keys for the IBM

General Purpose Keys

Flip	Control-F
Up	Control-U
Down	Control-D
Quit, leave page	Esc
Stop	Control-Break
Help	Fn-0

Word-Processing Keys

Select	Fn-1
Cut	Fn-2
Copy	Fn-3
Paste	Fn-4
Erase to End of Line	Fn-6
Next Screen	PgDn
Previous Screen	PgUp
Top of Page	Home
Bottom of Page	End
Beginning of Line	Control <—
End of Line	Control —>
Open a Line	Insert

Shapes Page Keys

Quit, leave page	Esc
Flip	Control-F

On the flip side of the page only:

Next Screen (Next Shape)	PgDn
Previous Screen (Previous Shape)	PgUp
Up, Down, Left, Right	Arrow keys
Cut	Fn-2
Copy	Fn-3
Paste	Fn-4
Make or Erase Block	Space Bar

Graphics Keys

Turtle-Move	Fn-9
Label	Fn-8
Leave Label or Turtle-Move	Esc

Contents Keys

Top of Page	Home
Bottom of Page	End
Remove a Page (Erase to End of Line)	Fn-6

Summary of *LogoWriter* Keys for the Macintosh

General Purpose Keys

Flip	⌘-F
Up	⌘-U
Down.	⌘-D
Quit, leave page	Esc, Clear, or ⌘-Shift
Stop	⌘-.

Word-Processing Keys

Select	⌘-1
Cut	⌘-2 or ⌘-X
Copy	⌘-3 or ⌘-C
Paste	⌘-4 or ⌘-V
Erase to End of Line	⌘-6
Next Screen	⌘—>
Previous Screen	⌘<—
Top of Page	⌘-Up Arrow
Bottom of Page	⌘-Down Arrow
Beginning of Line	⌘-B
End of Line	⌘-E
Open a Line	⌘-O

Shapes Page Keys

Quit, leave page	Esc
Flip	⌘-F

On the flip side of the page only:

Cut	⌘-2 or ⌘-X
Copy	⌘-3 or ⌘-C
Paste	⌘-4 or ⌘-V

Graphics Keys

Turtle-Move	⌘-9
Label	⌘-8
Leave Label or Turtle-Move	Esc

Appendix B
Quick Reference

LogoWriter Primitives and Special Words

* Primitive is a reporter (operator).
\# Primitive is either a command or a reporter.

Screen
- CC—Clear Command center
- CG—Clear Graphics
- CP—Clear Page
- CT—Clear Text

Scrapbook
- CLEARTOOLS
- CONTENTS
- DOS (IBM only)
- ERPAGE *pagename*—ERase Page
- FLIP
- * FRONT?
- GETPAGE (GP) *pagename*
- GETSHAPES *pagename* (Mac Only)
- GETSOMESHAPES *pagename list* (Mac Only)
- GETTOOLS *pagename*
- LEAVEPAGE
- LOAD *pagename*
- LOCK (Mac only)
- NAMEPAGE (NP) *pagename*
- NEWPAGE
- * PAGELIST
- RESTORE
- SAVEPAGE
- SHAPES
- * TOOLLIST
- UNDO (IBM and Apple only)
- UNLOCK

Workspace
- RECYCLE
- * SPACE

Timing
- CLOCK (Mac Only)
- RESET (Mac Only)
- TIMER (Mac Only)
- WAIT *number*

Sound
- ERSOUND *name* (Mac Only)
- PLAY *name* (Mac Only)
- RECORD *name number* (Mac Only)
- SOUNDLIST (Mac Only)
- TONE *frequency time*

Input/Output
- * BUTTON? *button number* (Mac—no input)
- CLEARCOM (Mac Only)
- * KEY?
- PADDLE *number* (IBM and Apple only)
- * READCHAR (Mac only)
- * READLIST (RL)
- * READLISTCC (RLCC)
- RECEIVECHAR (RCCC)
- SEND *word / list* (Mac only)
- SETBAND *number* (Mac only)
- SETCF *word1 word2* (Mac only)

Assigning
- CLEARNAME *word / list*
- CLEARNAMES
- MAKE *name word / list*
- NAME *word / list name*
- * NAME? *word*
- PRINTNAMES
- SHOWNAMES
- THING *names*

Printer Commands
- DSPACE—Double SPACE (IBM and Apple only)
- PRINTSCREEN
- PRINTTEXT
- PRINTTEXT80 (IBM and Apple only)
- SSPACE—Single SPACE (IBM and Apple only)

Special Words
END
FALSE
HELP *pagename* (Mac only)
STARTUP
TO
TRUE

System Modifying Primitives (IBM/Apple)
.BLOAD *name/pathname address*
.BSAVE *name/pathname address length*
.CALL *address*
.DEPOSIT *address byte*
* .EXAMINE *address*

Flow of Control/Logic
* AND *true/false1 true/false2*
CAREFULLY *list1 list2*
ERRORMESSAGE (Mac Only)
ERRORNUMBER (Mac Only)
* IF *true/false list to run*
\# IFELSE *true/false list.to.run1 list.to.run2*
LOCAL *word/list* (Mac Only)
* NOT *true/false*
* OR *true/false1 true/false2*
OUTPUT (OP) *word/list*
REPEAT *number list.to.run*
RUN *list.to.run*
STOP
STOPALL

Math (Note: all are reporters)
\+ - * / (Mac only)
> < =
<> <= >= \ ^
ABS *number* (Mac only)
ARCTAN *number*
COS *degrees*
DIFFERENCE *number1 number2* (Mac Only)
E (Mac Only)
EXP *number* (Mac Only)
GREATER? *number1 number2* (Mac Only)
INT *number*
LESS? *number1 number2* (Mac Only)
LN *number* (Mac Only)
LOG *number1 number2* (Mac Only)
PI (Mac Only)
POWER *number1 number2* (Mac Only)
PRODUCT *number1 number2* (Mac Only)
QUOTIENT *number1 number2* (Mac Only)
RERANDOM (Mac Only)
RANDOM *number*
ROUND *number*
SETNF *number* (Mac Only)
SETPOINT (Mac Only)
SETCOMMA (Mac Only)
SIN *degrees*
SQRT *number*
SUM *number1 number2* (Mac Only)
TAN *number*

Events
CLEAREVENTS
PRINTEVENTS (Mac Only)
SHOWEVENTS (Mac Only)
WHEN *letter list.to.run*

Graphics
* ALL
\# ASK *turtle/turtle.list list.to.run*
BACK (BK) *number*
* BG
CHANGECOLOR *number list of three numbers* (GS)
* CHARUNDER - CHARacter UNDER
CLEAN
* COLOR
* COLORUNDER
* COLORVALUE *number* (GS)
* CURSORPOS
* DISTANCE *list of two numbers* (GS/IBM/Mac)
EACH *list.to.run*
FASTTURTLE (GS/Mac)
FILL
FORWARD (FD) *number*
HEADING
HOME
HT—Hide Turtle
LABEL *word/list*
LEFT (LT) *number*
MOUSEPOS (Mac Only)
PD—Pen Down
PE—Pen Erase
* POS
PU—Pen Up
PX—Pen Reverse
RESETCOLORS (GS)
RG—Reset Graphics
RIGHT (RT) *number*
SETBG *number*
SETC *number*
SETH *number*
SETPOS *[x y]*

SETSH *number*
SETX *number*
SETY *number*
SHADE
* SHAPE
SLOWTURTLE (GS/Mac)
ST—Show Turtle
STAMP
TELL *turtle / turtle.list*
TOUCHING? (Mac Only)
* TOWARDS *list*
* WHO
WINDOW
WRAP
XCOR
YCOR

Text Editing/Words and Lists

* ASCII *char*
BOTTOM
* BUTFIRST (BF) *word / list*
* BUTLAST (BL) *word / list*
CB—Cursor Back
CD—Cursor Down
CF—Cursor Forward
* CHAR
* CLIPBOARD
COPY
* COUNT
CU—Cursor Up
CURSORCHAR
CUT
DELETE
* EMPTY? *word / list*
EOL
* EQUAL? *word / list1 word / list2*
* FIRST *word / list*
* FOUND?
* FPUT *word / list list*
* IDENTICAL? *word / list1 word / list2*
INSERT *word / list*
* ITEM *number word / list*
* LAST *word / list*
* LIST *word / list1 word / list2*
* LIST? *word / list*
* LPUT *word / list list*
* MEMBER? *word / list1 word / list2*
NEXTSCREEN
* NUMBER?
PARSE *word*
PASTE
PRESCREEN
PRINT (PR) *word / list*
REPLACE *word1 word2*
SEARCH *word*
SELECT
* SELECTED
* SENTENCE (SE) *word / list1 word / list2*
SETEXTPOS *number*
SETTC *number* (GS)
SHOW *word / list*
SOL—Start Of Line
TAB
* TC (GS)
* TEXTLEN
* TEXTPOS
TOP3
TYPE *word / list*
UNSELECT
* WORD *word1 word2*
* WORD? *word / list*

Disk (Apple or IBM)

* DISK
SETDISK *letter*

ProDos Primitives

BYE
COPYFILE *name / pathname1 name / pathname2*
CREATEDIR *pathname*
* DIRECTORIES
DISK (Mac only)
ERASEFILE *name / pathname*
* FILELIST
LOADPIC *name / pathname*
LOADTEXT *name / pathname*
* ONLINE
* PREFIX
RENAME *name / pathname1 name / pathname2*
SAVEPIC *name / pathname*
SAVETEXT *name / path*
SETDISK *word*
SETPREFIX *pathname*
SETSLOT
* SLOT
* .VERSION

IBM DOS Primitives

CHDIR *name / pathname*
COPYFILE *name / pathname1 name / pathname2*
* CURRENTDIR
* DIRECTORIES
DISK
DOS

ERASEFILE *name / pathname*
* FILELIST
LOADPIC *name / pathname*
LOADTEXT *name / pathname*
MKDIR *pathname*
RENAME *name / pathname1 name / pathname2*
RMDIR
SAVEPIC *name / pathname*
SAVETEXT *name / pathname*
SETDISK *word*
* .VERSION

Macintosh Primitives

COPYFILE *name / pathname1 name / pathname2*
CREATEDIR *pathname*
* DIRECTIONS
* DISKSPACE
ERASEFILE *filename*
* FILELIST
LOADPIC *filename / pagename*
LOADTEXT *filename / pagename*
* ONLINE
PAGE
* PREFIX
QUIT
RENAME *name / pathname1 name / pathname2*
RMDIR *name*
SAVEPIC *name / pathname*
SAVETEXT *filename*
.VERSION

Appendix C
Additional Primitives for the Macintosh

Math Primitives

number/ \number
\ is the equivalent of REMAINDER

ABS *number*
ABS reports the absolute value of its input.

DIFFERENCE *number1 number2*
DIFFERENCE is the prefix form of the subtraction sign (-). It takes two numbers as input and reports the result of subtracting the second input from the first one.

E
Reports the value of the transcendental number e.

EXP *number*
EXP takes one number as input. It reports the number e raised to the power of the input to EXP. This is the inverse of LN.

LN *number*
LN reports the natural logarithm of its input. This is the inverse of EXP.

LOG *number (or number1 number2)*
LOG reports the logarithm of its input in base 10.

PI
Reports the value of π.

POWER *number1 number2*
Reports the value of the first input raised to the second input.

PRODUCT *number1 number2*
This is the prefix form of the multiplication sign (*). It reports the product of its two inputs.

QUOTIENT *number1 number2*
QUOTIENT reports the result of dividing the first input by the second input. Division by zero gives an error. This is the prefix form of the division sign (/).

SETNF *number*
This primitive takes one input and sets the number of digits displayed after the decimal point. The default is for *LogoWriter* to display 4 digits. SETNF does not affect the precision of Macintosh *LogoWriter*. It always uses 16 digits for

computations. Macintosh *LogoWriter* can also display numbers in scientific notation. For example 1.0E10 means 10^{10}.

SETPOINT
Causes LogoWriter to use a decimal to separate the integer part from the fractional part of a number.

SETCOMMA
Causes LogoWriter to use a comma to separate the integer part from the fractional part of a number.

SUM *number1 number2*
This is the prefix form of the addition sign (+). It reports the sum of the two input numbers.

TAN *number*
TAN reports the tangent of its input in degrees.

Flow of Control and Logic

CAREFULLY *list1 list2*
CAREFULLY takes two lists as inputs. The first input is a list of instructions that is run. If there is an error while this list is running, the second list is run. If there is no error in the first list, the second is ignored and ERRORNUMBER is set to 0 while ERRORMESSAGE is an empty word.

ERRORMESSAGE
This reporter returns the most recent error message.

ERRORNUMBER
This reporter returns the number of the most recent error message.

-H
Pressing this key combination causes the current procedure to pause. Pressing any key causes it to begin again.

Time

CLOCK
This reporter returns a list containing the date and the time in the format [10/19/89 4:30:55 PM]

RESETT
Resets the timer to 0.

TIMER
This primitive reports the duration, in 60ths of a second, since the last RESETT command.

Help

HELP
The HELP primitive allows you to create your own Help page. It takes one input, the name of a page. The page can be looked at but not changed.

Help Menu
Under the Apple Menu at the left edge of the menu bar are two choices for help. The "Help Page..." option gets a page named help. The "Help Primitives..." displays a list of all the primitives in Macintosh *LogoWriter* in alphabetical order. Clicking on one of the primitives in the list causes a description of that primitive to appear.

Sound

ERSOUND *sound*
This command takes the name of a sound resource as input and erases it.

PLAY *sound*
This command takes one input and plays the named sound resource.

RECORD *sound duration*
This command takes two inputs, a name and a number. The first input is the name to be used to save a recorded sound. The second is the number of seconds to be recorded. This command works with Macintosh models with built in microphones.

SOUNDLIST
Reports the list of current sound resources.

Communication

CLEAR COM
Clears the communication line.

RECEIVECHAR, RECC
This reporter returns the character being received through the communication channel. If no character is being received, LogoWriter waits for a character.

SEND *word/list*
This command sends a word or a list through the communication line. It does not send a carriage return following the input.

SETBAUD *number*
This command takes a number as input. This number must be one of the following baud rates: 300, 1200, 2400, 4800, 9600, 19200. The default value is 4800.

SETCF
This command takes two inputs and sets the communication format. The first input is the category; the second input is the value. Here are the possible values:

Miscellaneous

BUTTON?
Reports true if mouse button has been clicked.

MOUSE POS
Returns the position of the mouse in turtle coordinates.

PRIMITIVES
This reports a list of all of the primitives in LogoWriter.

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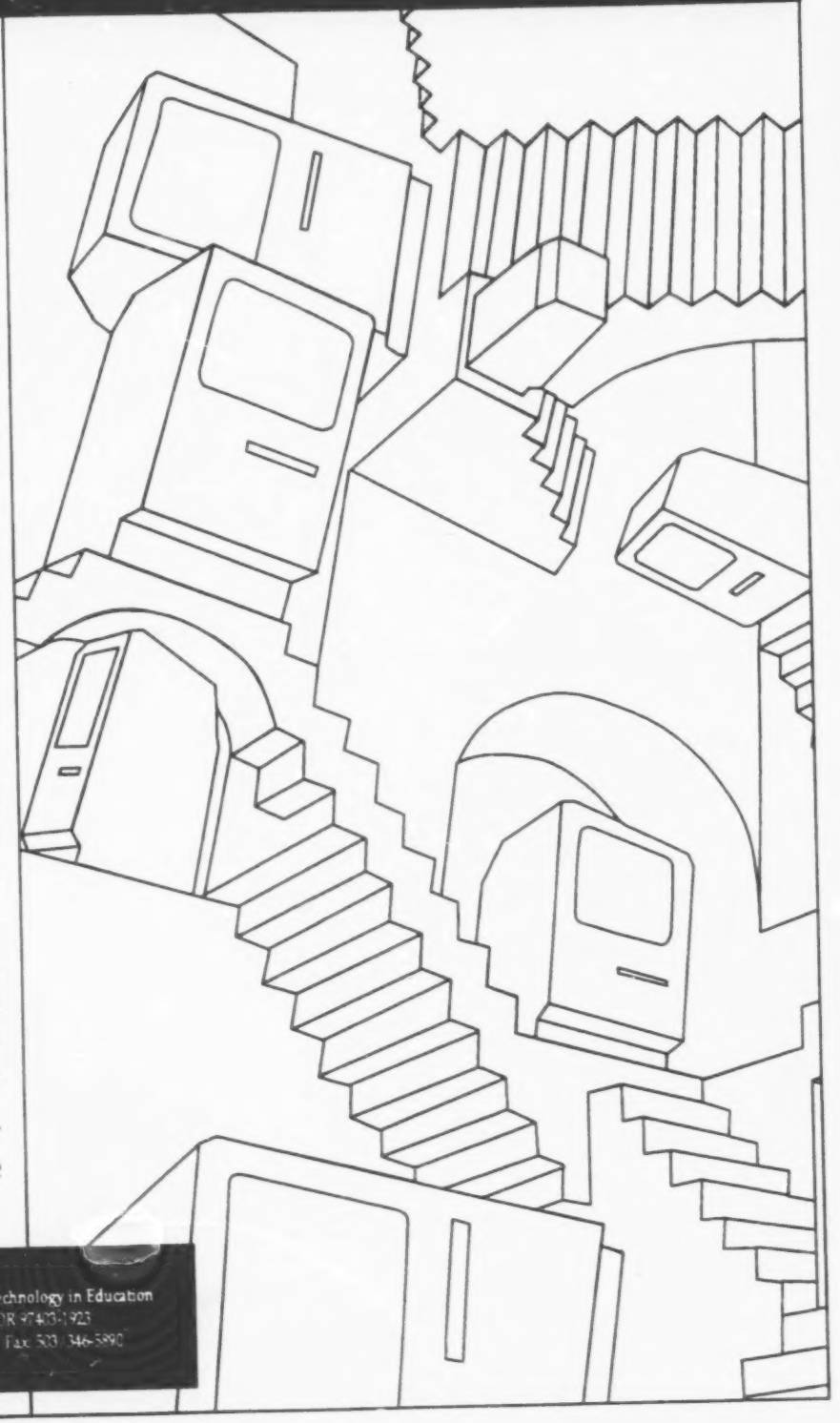

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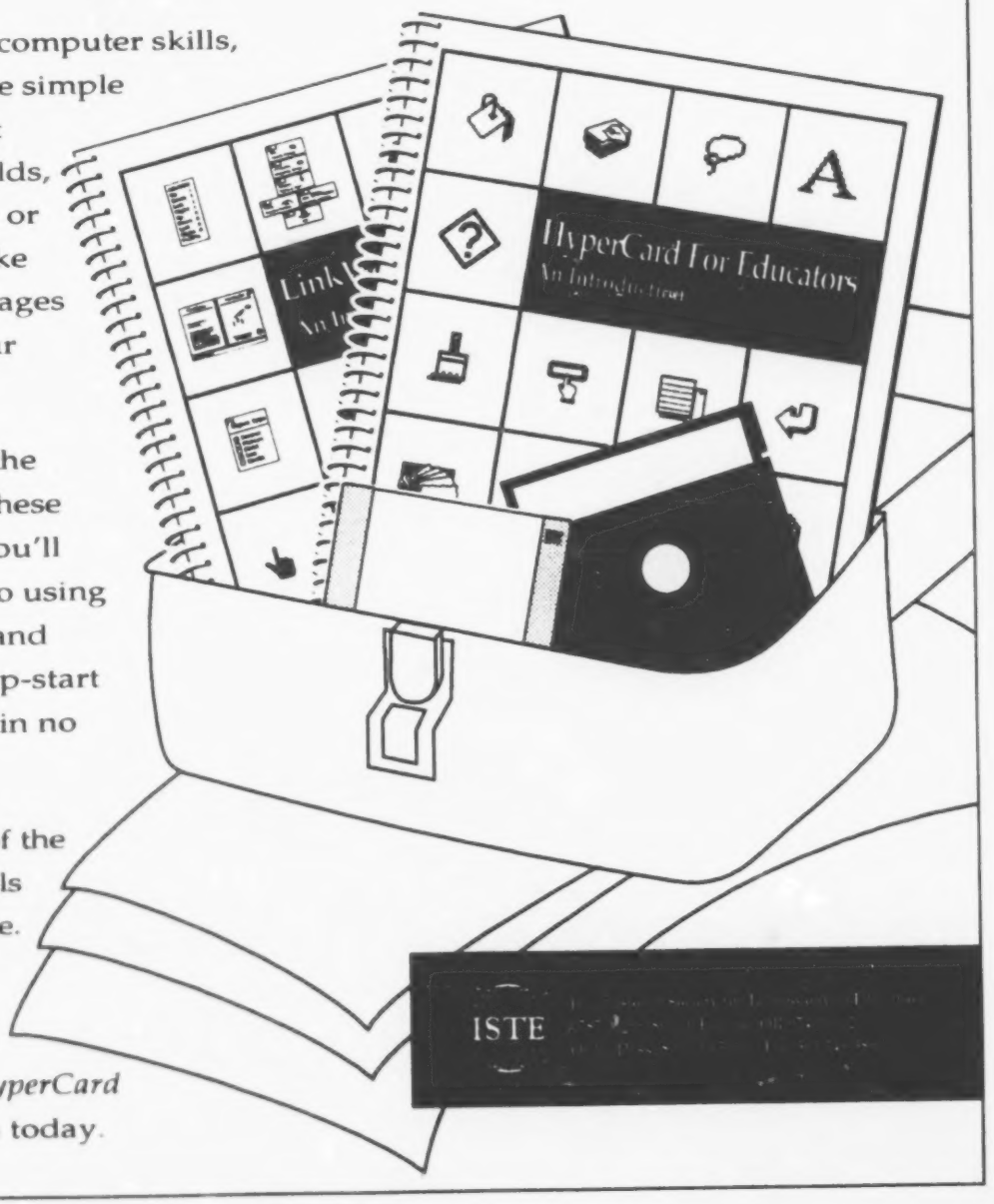
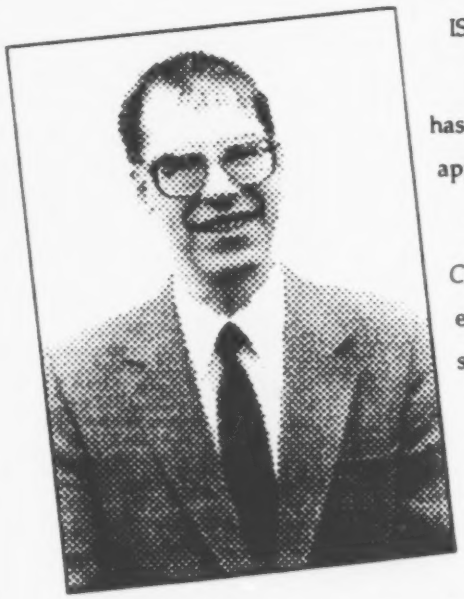


Dave Moursund knows...



ISTE Executive Officer Dave Moursund is dedicated to improving education.

Since he first began working with computers in education in 1963, Dave has constantly sought to improve education at all levels through the use of appropriate computer-related technology.

As a continuing part of this effort, Dave's newest book—*The Technology Coordinator*—discusses the goals of education and how teachers and teachers of teachers can join in the enormous change process facing today's schools.

This book was written for educators who want to play a leadership role in the instructional use of technology. Dave uses interviews with technology coordinators to illustrate the needs, roles, and qualifications needed to accomplish the goals of educational technology.

In recent years, Dave has focused much of his writing efforts on problem solving and on staff development. Some of Dave's other books also available from ISTE include *Problem-Solving Models for Computer Literacy, LogoWriter for Educators: A Problem Solving Approach, Logo PLUS for Educators: A Problem Solving Approach* (co-authored with Sharon Yoder), and *Effective Inservice for Integrating Computer-as-Tool into the Curriculum.*

"Our educational system is not nearly as good as it could be," says Dave. "We know that computer-related technology can make substantial contributions to improving both the content of the curriculum and the instructional delivery system. Such technology can empower students and teachers."

ISTE believes that computer-related technology must become a tool that students and teachers use routinely if students are to be adequately prepared for adult citizenship in our Information Age society.

ISTE's wide range of periodicals, publications, projects, and other activities mirrors this vision and commitment.

Call us today to order Dave's latest books, and discover how he and ISTE are leading the way for the educators and school systems of tomorrow.

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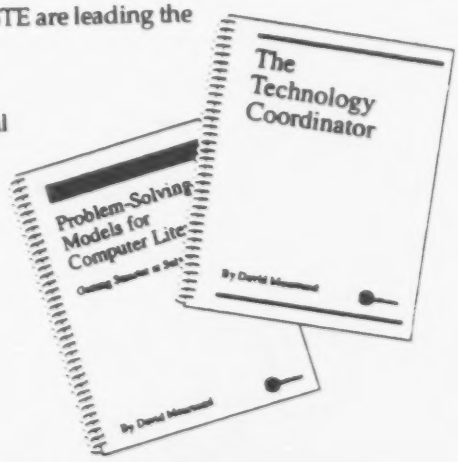

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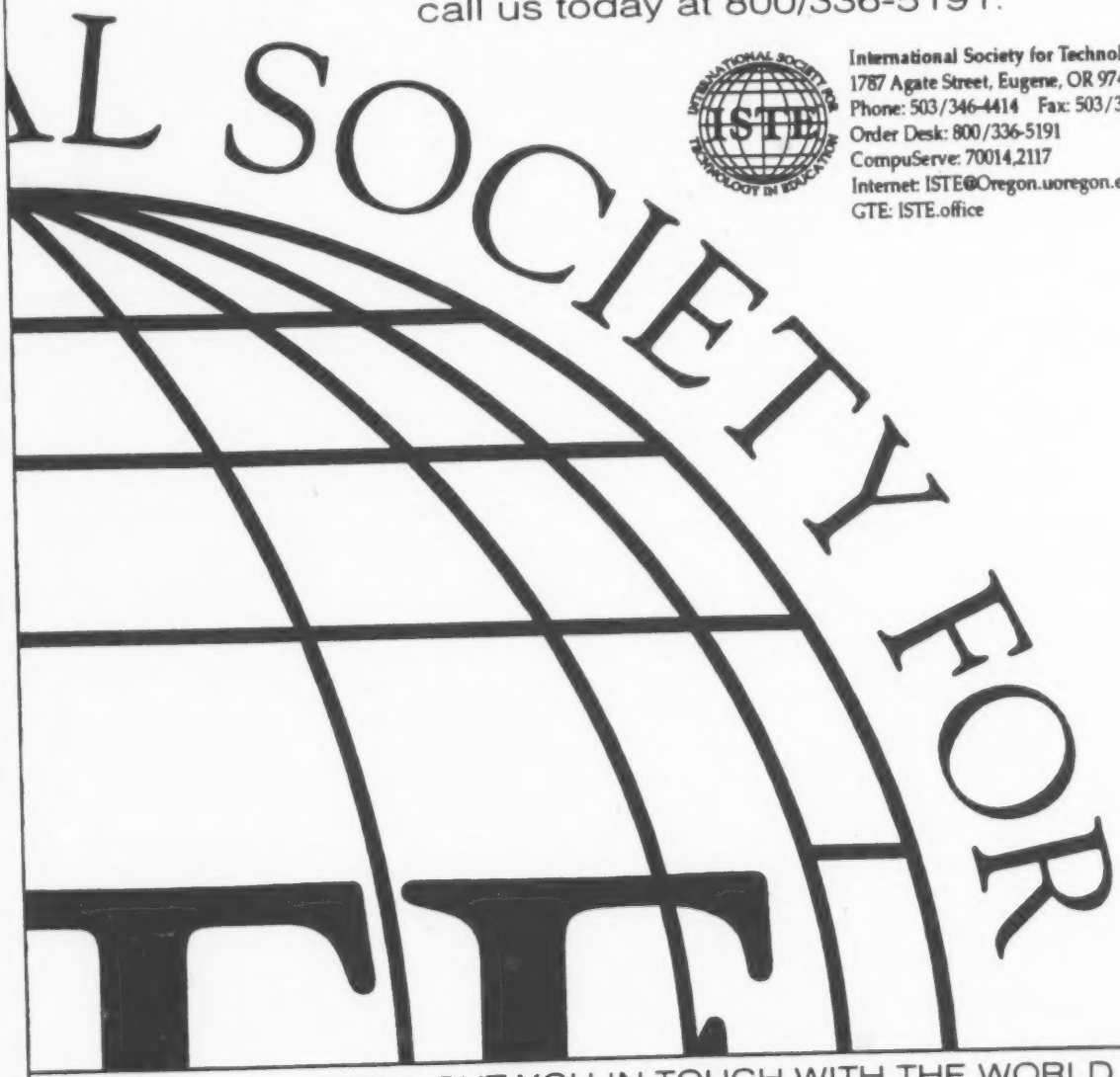

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